NUMBER THEORY AND LINEAR ALGEBRA
MM6B12

Objective Type Questions

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1. The rank of the identity matrix of order \( n \) is:
   (a) \( n - 1 \)
   (b) \( n \)
   (c) \( n + 1 \)

2. If \( A \) is a non-singular matrix of order \( n \), then the rank of \( A \) is:
   (a) \( n \)
   (b) 0
   (c) \( n - 1 \)

3. If \( A = \begin{bmatrix} 2 & 5 \\ 5 & 2 \end{bmatrix} \), then \( \rho(A) \) is:
   (a) 1
   (b) 2
   (c) 0

4. The rank of the matrix \( A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \) is:
   (a) 1
   (b) 2
   (c) 3

5. Rank of null matrix is:
   (a) 0
   (b) 1
   (c) Not defined

6. If two matrices \( A \) and \( B \) have the same size and the same rank, then
   (a) they have determinant 0
   (b) they are equivalent
   (c) \( A \) and \( B \) have common elements.

7. If the number of variables in a non-homogeneous system \( AX = B \) is \( n \), then the system possesses a unique solution if:
   (a) \( \rho(A) < \rho[A, B] \)
   (b) \( \rho(A) > \rho[A, B] \)
   (c) \( \rho(A) = \rho[A, B] = n \)

8. The system \( AX = 0 \) in \( n \) unknowns has non-trivial solution if:
   (a) \( \rho(A) > n \)
   (b) \( \rho(A) < n \)
   (c) \( \rho(A) = n \)

9. A system of \( m \) non-homogeneous linear equations \( AX = B \) in \( n \) unknowns is consistent if:
(a) \( \rho(A) = \rho[A, B] \)
(b) \( m = n \)
(c) \( \rho(A) \neq \rho[A, B] \)

10. The system \( AX = 0 \) in \( n \) unknowns has only trivial solution if:
   (a) \( \rho(A) > n \)
   (b) \( \rho(A) < n \)
   (c) \( \rho(A) = n \)

11. The system of equations \( 4x + 6y = 5, 8x + 12y = 10 \) has:
   (a) A unique solution
   (b) No solution
   (c) Infinitely many solutions

12. The system of equations \( 2x + 3y = 5, 6x + 9y = a \) has infinitely many solution if \( a \) is:
   (a) \( 2 \)
   (b) \( 15 \)
   (c) \( 6 \)

13. If \( A \) is a square matrix of order \( n \) and \( \lambda \) is a scalar, then the characteristic polynomial of \( A \) is obtained by expanding the determinant:
   (a) \( |\lambda A| \)
   (b) \( |\lambda A - I_n| \)
   (c) \( |A - \lambda I_n| \)

14. At least one characteristic roots of every singular matrix is equal to:
   (a) \( 1 \)
   (b) \( -1 \)
   (c) \( 0 \)

15. The characteristic roots of two matrices \( A \) and \( BAB^{-1} \) are:
   (a) The same
   (b) Different
   (c) Always zero

16. The scalar \( \lambda \) is a characteristic root of the matrix \( A \) if:
   (a) \( A - \lambda I \) is non-singular
   (b) \( A - \lambda I \) is singular
   (c) \( A \) is singular

17. If eigenvalue of matrix \( A \) is \( \lambda \), then eigenvalue of \( P^{-1}AP \) is:
   (a) \( 1 \)
   (b) \( \frac{1}{\lambda} \)
18. The product of all characteristic roots of a square matrix $A$ is equal to:
   (a) 0
   (b) 1
   (c) $|A|$

19. If eigenvalue of matrix $A$ is $\lambda$, then eigenvalue of $A^2$ is:
   (a) 1
   (b) $\frac{1}{\lambda}$
   (c) $\lambda^2$

20. If $A$ is invertible matrix and eigenvalue of $A$ is $\lambda$, then eigenvalue of $A^{-1}$ is:
   (a) 1
   (b) $\frac{1}{\lambda}$
   (c) $\lambda$

21. If the determinant of a matrix $A$ is non zero, then its eigenvalues of $A$ are:
   (a) 1
   (b) 0
   (c) Non zero

22. If the determinant of a matrix $A$ is zero, then one of its eigenvalues of $A$ is:
   (a) 1
   (b) 0
   (c) -1

23. Given integers $a$ and $b$, with $b > 0$, then
   (a) $a = qb + r$ $0 \leq r < b$
   (b) $a = qb + r$ $0 \leq r < 1$
   (c) $a = qb + r$ $0 \leq r < a$

24. If $a|1$, then
   (a) $a = \pm 1$
   (b) $a = 0$
   (c) $a = 1$

25. If $a|b$ and $b|a$ then
   (a) $a = b$
   (b) $a = \pm b$
   (c) $a = b = 0$

26. If $a|b$ and $a|c$ then
   (a) $a|bx + cy$
(b) $a|1$
(c) $a = 1$

27. If $a|b$ with $b \neq 0$, then
   (a) $|a| = |b|$
   (b) $|a| > |b|$
   (c) $|a| \leq |b|$

28. Let $g c d (a, b) = d$, then
   (a) $d|a$ only
   (b) $d|b$ only
   (c) $d|a$ and $d|b$

29. Let $g c d (a, b) = d$. If $c|a$ and $c|b$, then
   (a) $c \leq d$
   (b) $c \geq d$
   (c) $c = 1$

30. If $a$ and $b$ are relatively prime then
   (a) $a|b$
   (b) $b|a$
   (c) $g c d (a, b) = 1$

31. Let $g c d (a, b) = d$, then
   (a) $d = ax + by$
   (b) $a|d$
   (c) $b|d$

32. Let $g c d (a, b) = d$, then $g c d \left(\frac{a}{d}, \frac{b}{d}\right) =$
   (a) $1$
   (b) $a$
   (c) $b$

33. If $a|c$ and $b|c$ with $g c d (a, b) = 1$, then
   (a) $ab|c$
   (b) $(a + b)|c$
   (c) $(a - b)|c$

34. If $a|bc$, with $g c d (a, b) = 1$, then
   (a) $a|1$
   (b) $a|c$
   (c) $a \nmid c$

35. Let $g c d (a, b) = d$. If $c|a$ and $c|b$, then
(a) \(c \mid d\)
(b) \(c \nmid d\)
(c) \(c = 1\)

36. If \(a\) and \(b\) are given integers, not both zero, then the set \(T = \{ ax + by : x, y \text{ are integers} \}\) contains:

(a) Multiples of \(d\)
(b) Divisors of \(d\)
(c) Divisors of \(a\) and \(b\)

37. Let \(a\) and \(b\) be integers, not both zero. Then \(a\) and \(b\) are relatively prime if and only if

(a) \(1 = ax + by\), for all integers \(x\) and \(y\).

(b) \(1 = ax + by\), for some integers \(x\) and \(y\).

(c) \(1 = ax + by\), for unique integers \(x\) and \(y\).

38. If \(k > 0\), then \(\gcd(ka, kb)\) is

(a) \(k \gcd(a, b)\).
(b) \(\gcd(a, b)\).
(c) \(1\)

39. If \(k \neq 0\), then \(\gcd(ka, kb)\) is

(a) \(|k| \gcd(a, b)\).
(b) \(k \gcd(a, b)\).
(c) \(1\)

40. \(\gcd(12, 30) = 3 \quad \ldots \ldots \ldots \ldots \ldots \ldots \)

(a) \(\gcd(12, 10)\)
(b) \(\gcd(4, 30)\)
(c) \(\gcd(4, 10)\)

41. If the least common multiple of two nonzero integers \(a\) and \(b\), is \(m\), then

(a) \(m \mid a \text{ and } m \mid b\)

(b) \(a \mid m \text{ and } b \mid m\)

(c) \(m = ab\)

42. If the least common multiple of two nonzero integers \(a\) and \(b\), is \(m\). If \(a \mid c\) and \(b \mid c\), with

\(c > 0\), then

(a) \(m = c\)

(b) \(m > c\)

(c) \(m \leq c\)

43. For positive integers \(a\) and \(b\), \(\gcd(a, b)\), \(\lcm(a, b)\) is

(a) \(1\)

(b) \(0\)

(c) \(ab\)
44. For any choice of positive integers \( a \) and \( b \), \( \text{lcm}(a, b) = ab \) if and only if
   (a) \( \gcd(a, b) = 1 \).
   (b) \( \gcd(a, b) = ab \).
   (c) \( \gcd(a, b) = a + b \).

45. The linear Diophantine equation \( ax + by = c \) has a solution if and only if
   (a) if \( \text{d} \mid c \), where \( \text{d} = \gcd(a, b) \)
   (b) \( c = 1 \)
   (c) \( \gcd(a, b) = 1 \).

46. An integer greater than 1 that is not a prime is termed
   (a) Even number
   (b) Odd number
   (c) Composite number

47. If \( p \) is a prime and \( p \mid ab \), then
   (a) \( p \mid a \) or \( p \mid b \).
   (b) \( p \mid a \) and \( p \mid b \).
   (c) \( p \mid a \).

48. If \( p \) is a prime and \( p \mid a_1 a_2 \cdots a_n \), then
   (a) \( p = a_k \) for some \( k \)
   (b) \( p \mid a_k \) for some \( k \)
   (c) \( p = 2 \).

49. If \( p, q_1, q_2, \ldots, q_n \) are all primes and \( p \mid q_1 q_2 \cdots q_n \), then
   (a) \( p = q_k \) for some \( k \)
   (b) \( p = 2 \)
   (c) \( q_k = 2 \) for some \( k \).

50. Any positive integer \( n > 1 \), the canonical form for \( n \) is:
   (a) \( n = p_1^{k_1} p_2^{k_2} \cdots p_r^{k_r} \), where \( p_i \)'s are primes
   (b) \( n = p_1 p_2 \cdots p_r \), where \( p_i \)'s are primes
   (c) \( n = 2p + 1 \), where \( p \) is a prime.

51. The number \( \sqrt{2} \) is:
   (a) Rational
   (b) Irrational
   (c) Not a real number

52. 509 is:
   (a) Composite
   (b) Prime
   (c) even
53. $5^# + 1 =$
   (a) 31
   (b) 1000
   (c) 21

54. If $P_n$ is the $n^{th}$ prime number, then
   (a) $P_n = n + 1$
   (b) $P_n \leq 2^{2n-1}$
   (c) $P_n = n! + 1$

55. For $n \geq 1$, there are at least $\ldots \ldots$ primes less than $2^{2n}$
   (a) $n$
   (b) $n - 1$
   (c) $n + 1$

56. If $a \equiv b \pmod{n}$, then
   (a) $n|a$ and $n|b$
   (b) $n|b$ only
   (c) $n|a - b$

57. If $a \equiv b \pmod{n}$, then
   (a) $a$ and $b$ leave the same nonnegative remainder when divided by $n$.
   (b) $a$ and $b$ leave the different nonnegative remainder when divided by $n$.
   (c) $a$ and $b$ need not leave the same nonnegative remainder when divided by $n$.

58. $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$, then
   (a) $a \equiv c \pmod{n}$
   (b) $a = b$
   (c) $a = c$

59. If $ca \equiv cb \pmod{n}$ and $\gcd(c, n) = d$ then
   (a) $a \equiv b \pmod{n/d}$
   (b) $a \equiv b \pmod{n}$
   (c) $a \equiv b \pmod{d/n}$

60. If $ca \equiv cb \pmod{n}$ and $\gcd(c, n) = 1$ then
   (a) $a \equiv b \pmod{n/c}$
   (b) $a \equiv b \pmod{n}$
   (c) $a \equiv b \pmod{c/n}$

61. If $a \equiv b \pmod{n}$, then
   (a) $a - b = n$
   (b) $a - b = kn$, for some integer $k$.
   (c) $a + b = kn$, for some integer $k$. 
62. If \( ca \equiv cb \pmod{p} \) and \( p \nmid c \), where \( p \) is a prime number, then
   (a) \( a \equiv b \pmod{p/c} \)
   (b) \( a \not\equiv b \pmod{p} \)
   (c) \( a \equiv b \pmod{p} \)

63. If \( a \equiv b \pmod{n} \), and \( m|n \), then
   (a) \( a \equiv b \pmod{m} \)
   (b) \( a \not\equiv b \pmod{m} \)
   (c) \( a \equiv b \pmod{\frac{n}{m}} \)

64. If \( a \equiv b \pmod{n} \), and \( c > 0 \), then
   (a) \( ca \equiv cb \pmod{cn} \)
   (b) \( ca \not\equiv cb \pmod{cn} \)
   (c) \( a \equiv b \pmod{\frac{cn}{c}} \)

65. If \( a \equiv b \pmod{n} \) and \( \gcd(a, n) = d \), then \( \gcd(b, n) \) is:
   (a) \( a \)
   (b) \( d \)
   (c) \( nd \)

66. Let \( P(x) = \sum_{i=0}^{n} c_i x^i \) be a polynomial function of \( x \) with integral coefficients \( c_k \). If \( a \equiv b \pmod{n} \), then
   (a) \( a \equiv P(b) \pmod{n} \)
   (b) \( P(a) \equiv P(b) \pmod{n} \)
   (c) \( P(a) \equiv b \pmod{n} \)

67. If \( a \equiv b \pmod{n} \) and \( a \) is a solution of \( P(x) \equiv 0 \pmod{n} \), then
   (a) \( b \) is also a solution
   (b) \( b \) need not be a solution
   (c) \( 0 \) is a solution

68. A positive integer is divisible by 9 if and only if the sum of the digits in its decimal representation is divisible by
   (a) \( 3 \)
   (b) \( 81 \)
   (c) \( 9 \)

69. A positive integer is divisible by 11 if and only if the alternating sum of its digits is divisible by
   (a) \( 121 \)
   (b) \( 22 \)
   (c) \( 11 \)
70. The linear congruence $ax \equiv b \pmod{n}$ has a solution if and only if
   (a) $b = 1$
   (b) $b = 0$
   (c) $d | b$, where $d = \gcd(a, n)$

71. If $\gcd(a, n) = 1$, then the congruence $ax \equiv b \pmod{n}$ has
   (a) Infinitely many solutions modulo $n$
   (b) Unique solution modulo $n$
   (c) More than one solution modulo $n$

72. The linear congruence $18x \equiv 6 \pmod{3}$ has
   (a) Infinitely many solutions modulo $3$
   (b) Unique solution modulo $3$
   (c) Exactly 3 solution modulo 3

73. The linear congruence $19x \equiv 6 \pmod{30}$ has
   (a) Infinitely many solutions modulo 30
   (b) Unique solution modulo 30
   (c) Exactly 6 solution modulo 30

74. The system of linear congruences $ax + by \equiv r \pmod{n}$ and $cx + dy \equiv s \pmod{n}$ has a
   unique solution modulo $n$ whenever:
   (a) $\gcd(ad - bc, n) = 1$.
   (b) $\gcd(ad, bc) = 1$
   (c) $\gcd(ad, bc) = n$

75. Let $p$ be a prime and suppose that $p \nmid a$. Then:
   (a) $a^p \equiv 1 \pmod{p}$
   (b) $a^{p-1} \equiv 1 \pmod{p}$
   (c) $a^{p-1} \equiv -1 \pmod{p}$

76. If $p$ is a prime, then for any integer $a$:
   (a) $a^p \equiv a \pmod{p}$
   (b) $a^{p-1} \equiv 1 \pmod{p}$
   (c) $a^{p-1} \equiv -1 \pmod{p}$

77. If $p$ is a prime, then
   (a) $(p - 1)! \equiv 1 \pmod{p}$
   (b) $(p - 1)! \equiv -1 \pmod{p}$
   (c) $(p - 1)! \equiv 0 \pmod{p}$

78. $\sigma(12)$ is
   (a) 28
79. \( \tau(12) \) is
(a) 28
(b) 5
(c) 6

80. \( \tau(n) = 2 \) if and only if
(a) \( n \) is a prime number
(b) \( n \) is an odd number
(c) \( n \) is an even number

81. \( \sigma(n) = n + 1 \) if and only if
(a) \( n \) is an odd number
(b) \( n \) is an even number
(c) \( n \) is a prime number

82. If \( n = p_1^{k_1} p_2^{k_2} \ldots p_r^{k_r} \) is the canonical form for \( n > 1 \), then
(a) \( \tau(n) = k_1 + k_2 + \cdots + k_r \)
(b) \( \tau(n) = (k_1 + 1)(k_2 + 1) \ldots (k_r + 1) \)
(c) \( \tau(n) = (k_1 - 1)(k_2 - 1) \ldots (k_r - 1) \)

83. If \( n = p_1^{k_1} p_2^{k_2} \ldots p_r^{k_r} \) is the canonical form for \( n > 1 \), then
(a) \( \sigma(n) = k_1 + k_2 + \cdots + k_r \)
(b) \( \sigma(n) = \frac{p_1^{k_1+1} - 1}{p_1 - 1} \frac{p_2^{k_2+1} - 1}{p_2 - 1} \ldots \frac{p_r^{k_r+1} - 1}{p_r - 1} \)
(c) \( \sigma(n) = (k_1 + 1) + (k_2 + 1) + \cdots + (k_r + 1) \)

84. A number-theoretic function \( f \) is said to be multiplicative if \( f(m, n) = f(m)f(n) \):
(a) Whenever \( \text{lcm}(m, n) = mn \)
(b) For all integers \( m \) and \( n \)
(c) For all positive integers \( m \) and \( n \)

85. A number-theoretic function \( f \) is said to be multiplicative if \( f(m, n) = f(m)f(n) \):
(a) For all integers \( m \) and \( n \)
(b) For all positive integers \( m \) and \( n \)
(c) Whenever \( \text{gcd}(m, n) = 1 \)

86. A number-theoretic function \( f \) is said to be multiplicative if \( f(m, n) = f(m)f(n) \):
(a) For all integers \( m \) and \( n \)
(b) For all positive integers \( m \) and \( n \)
(c) For all relatively prime integers.

87. The functions \( \tau \) and \( \sigma \) are both multiplicative functions. The statement is:
(a) False
(b) True
(c) Partially true

88. The Euler’s Phi- function is:
   (a) **Multiplicative**
   (b) Not Multiplicative
   (c) Injective

89. Which of the following statement is true:
   (a) The functions $\tau$ and $\sigma$ are both multiplicative functions
   (b) The Euler’s Phi- function is injective
   (c) The Euler’s Phi- function is not Multiplicative

90. For $n > 2$, $\phi(n)$ is:
   (a) Prime number
   (b) *Even number*
   (c) Odd number

91. If $n > 1$, is prime. Then $\phi(n)$ is:
   (a) $n - 1$
   (b) $n$
   (c) $n + 1$

92. $\phi(13)$ is:
   (a) 13
   (b) 12
   (c) 14

93. If $n \geq 1$ and $\gcd(a, n) = 1$, then
   (a) $a^{\phi(n)} \equiv -1 \pmod{n}$
   (b) $a^{\phi(n)} \equiv 0 \pmod{n}$
   (c) $a^{\phi(n)} \equiv 1 \pmod{n}$

94. For $n > 1$, the sum of the positive integers less than $n$ and relatively prime to $n$ is:
   (a) $\phi(n)$
   (b) $\frac{1}{2} \phi(n)$
   (c) $\frac{n}{2} \phi(n)$

95. If $n = p_1^{k_1}p_2^{k_2} \ldots p_r^{k_r}$ is the canonical form for $n > 1$, then
   (a) $\phi(n) = n \left(1 + \frac{1}{p_1}\right) \left(1 + \frac{1}{p_2}\right) \ldots \left(1 + \frac{1}{p_r}\right)$
   (b) $\phi(n) = n \left(1 - \frac{1}{p_1}\right) \left(1 - \frac{1}{p_2}\right) \ldots \left(1 - \frac{1}{p_r}\right)$
(c) $\phi(n) = \left(1 - \frac{1}{p_1}\right) \left(1 - \frac{1}{p_2}\right) \ldots \left(1 - \frac{1}{p_r}\right)$

96. If $p$ is a prime and $k > 0$, then:
(a) $\phi(p^k) = p^k \left(1 + \frac{1}{p}\right)$
(b) $\phi(p^k) = \left(1 - \frac{1}{p}\right)$
(c) $\phi(p^k) = p^k \left(1 - \frac{1}{p}\right)$

97. Given integers $a, b, c$, $\gcd(a, bc) = 1$ if and only if
(a) $\gcd(a, b) = 1$ and $\gcd(a, c) = 1$
(b) $\gcd(a, b) = 1$ and $\gcd(b, c) = 1$
(c) $\gcd(a, b) = 1$ and $\gcd(a, c) = b$

98. $\phi(2^3)$ is
(a) 2
(b) 3
(c) 7

99. Which of the following statement is false:
(a) If $p$ is a prime and $k > 0$, then $\phi(p^k) = \left(1 - \frac{1}{p}\right)$
(b) Given integers $a, b, c$, $\gcd(a, bc) = 1$ then $\gcd(a, b) = 1$ and $\gcd(a, c) = 1$
(c) $\phi$ is an oneto-one function

100. Which of the following statement is true:
(a) If $p$ is a prime and $k > 0$, then $\phi(p^k) = \left(1 + \frac{1}{p}\right)$
(b) Given integers $a, b, c$, $\gcd(a, bc) = 1$ then $\gcd(a, b) = 1$ and $\gcd(a, c) = 1$
(c) $\phi$ is an one-to-one function