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SCHOOL OF DISTANCE EDUCATION

STUDY MATERIAL
BA PHILOSOPHY

(2011 Admission)

V SEMESTER
CORE COURSE

ESSENTIALS OF SYMBOLIC LOGIC

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©
Reserved
<table>
<thead>
<tr>
<th>UNIT</th>
<th>CONTENT</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>INTRODUCTION</td>
<td>06-15</td>
</tr>
<tr>
<td></td>
<td>SYMBOLIC LOGIC &amp; CLASSICAL LOGIC</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>PROPOSITIONAL LOGIC</td>
<td>16-34</td>
</tr>
<tr>
<td></td>
<td>SIMPLE &amp; COMPOUND PROPOSITIONS</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>PREDICATE LOGIC</td>
<td>34-38</td>
</tr>
<tr>
<td></td>
<td>QUESTIONS</td>
<td>39 - 52</td>
</tr>
</tbody>
</table>
PY.V.B.07. ESSENTIALS OF SYMBOLIC LOGIC

Core course 07  4 Credits

Aim:

To equip the students with a knowledge of the basic concepts of modern logic as a development of classical logic.

Objectives:

(1) To introduce the modern classification of propositions and the branches of symbolic logic, and to show the relation between the two.

(2) To introduce propositional logic in detail stating the truth functional propositions, symbolization, truth tables, argument forms, statement forms, methods of proving validity / invalidity of arguments.

(3) To introduce predicate logic, stating quantification theory and symbolization of general propositions.
UNIT I - INTRODUCTION

SYMBOLIC LOGIC AND CLASSICAL LOGIC

Etymologically, logic is the science of logos. The Greek word logos means thought. There are many thought-processes such as reasoning, remembering, imagining. Reasoning is a thought process in which inference takes place. Logic is the science of reasoning. The logician is not concerned with the actual process of inference. The logician is concerned with the correctness of the completed process of inference.

Inference is a thought process in which one proposition is arrived at on the basis of other proposition or propositions. Corresponding to every inference, there is an argument. An argument consists of a group of propositions in which one proposition is claimed to follow from the other propositions providing grounds for the proposition. An argument consists of premises and conclusion.

The proposition arrived at is called the conclusion. The proposition or propositions on the basis of which the conclusion is arrived at is called premise or premises. Thus every argument has a structure.

Premises and conclusion are true or false. Argument is valid or invalid. Truth or falsity and validity or invalidity are to be distinguished.

Logic is the science of correct or good or valid reasoning. Logic is the study of the methods and principles used to distinguish between valid and invalid arguments. Every scientist has to see that his reasoning is in accordance with valid argument pattern. Such knowledge is provided by logic. The traditional logicians have regarded logic as the science of sciences.

In order to study the validity of reasoning or argument, the various forms of reasoning has to be determined, because validity depends on form of argument. The distinction between form and matter of an argument is to be understood. The matter of an argument is the subject-matter of with which the argument deals. The pattern or structure of the argument is the form of the argument. The form of the argument is determined by the manner in which it’s parts are related. Use of symbols reveals the form of arguments.

In the history of logic, Classical logic and Symbolic logic are two important stages of development. Classical logic is also called Aristotelian logic or Ancient logic/Traditional logic, in contrast to Symbolic logic or Mathematical logic or Modern logic.

Arguments presented in English or any other natural language are often difficult to appraise because of the vague and equivocal nature of words used, the ambiguity of their construction, the misleading idioms they may contain. To avoid those peripheral difficulties, it is convenient to set up an artificial symbolic language, free from such defects, in which statements and arguments can be
formulated. Symbolic logic has developed in recent past in connection with mathematical thinking and research.

Symbolic logic originated in connection with mathematical theory. Symbolic logic has a short history and the traditional or classical Aristotelian logic has a long one. But the difference between them is that of different stages of development. The Symbolic logic having a mathematical appearance involving heavy use of specialized technical symbols and minimum use of language, is the development and refinement of Aristotelian logic or classical logic. The Aristotelian logic contains germs of development in the manner and direction given to it by modern logicians. Classical logic and symbolic logic are interconnected. Classical logic is related to symbolic logic as embryo to adult organism.

According to Basson and O’ Conner, “modern symbolic logic is a development of the concepts and techniques which were implicit in the work of Aristotle”. It is now generally agreed by logicians that modern symbolic logic is a development of concepts and techniques which were implicit in the work of Aristotle.

The foundations of logic were laid by Aristotle in the fourth century B.C. that it seemed to most of Aristotle’s successors to be a finished science. But it is now realized that his treatment covered only a small, though important, branch of logic. The very thoroughness of his achievement was a part cause of the failure of logicians to make any significant contributions to the subject during the next 2000 years. Both the Greek successors of Aristotle and the medieval scholastics made several important logical discoveries. But the importance of these discoveries was not realized at the time they were made. The general belief that all the important logical discoveries have been made by Aristotle naturally tended to prevent philosophers from assessing any new discovery at its true value. The undeveloped state of the mathematical sciences prior to the seventeenth century was another important reason.

The difference between modern and classical logic is not one of kind but of degree. The greater extent to which modern logic has developed its own special technical language has made it more powerful a tool for analysis and deduction. The special symbols of modern logic help us to exhibit with greater clarity the logical structures of propositions and arguments whose forms may tend to be obscured by the unwieldiness of ordinary language. The value of the logician’s special symbols is the aid they give in the actual use and manipulation of statements and arguments. This is comparable to the replacement of Roman numerals by the Arabic notation. Arabic numerals are clearer and more easily comprehended than the older Roman numerals. Similarly, the drawing of inferences and the appraisal of arguments is greatly facilitated by the adoption of special logical notations.

Aristotle had introduced into logic the important notion of a variable. A variable is a symbol which can stand for any one of a given range of values. In the expression “x+2=5”, x is a variable. Aristotle’s use of variable s in logic was
restricted to representing the terms used in syllogistic arguments by letters of the alphabet, in order to bring out more clearly the logical structure of arguments.

The argument,

\[
\begin{align*}
\text{All men are mortal} \\
\text{Ram is a man} \\
\text{Therefore, Ram is mortal}
\end{align*}
\]

Is expressed, using symbols of the Aristotelian logic as follows:

\[
\begin{align*}
\text{All M is P} \\
\text{S is M} \\
\text{Therefore, S is P}
\end{align*}
\]

The symbols used in the example are called variables.

But the use of variables in symbolic logic is much wider than this. In symbolic logic, statement variables, and quantifiers are used to express the form of arguments.

A distinguished modern logician C.I. Lewis has cited three characteristics of symbolic logic:

1. **The use of ideograms.**

   Ideograms are signs which stand directly for concepts. For example, the multiplication sign (\(\times\)) or the question mark (?) are ideograms. In symbolic logic symbols which stand directly for concepts are used.

2. **The deductive method.**

   The characteristic of this method is that from a small number of statements we can generate, by the application of a limited number of rules, an indefinite number of other new statements.

3. **The use of variables.**

   In symbolic logic, variables have an extensive use. These three characteristics of symbolic logic are also characteristics of mathematics. Thus the development of symbolic logic has been bound up with the development of mathematics. All the pioneers of symbolic logic were either mathematicians or philosophers with a training in mathematical methods.

   The first important name in the development of modern symbolic logic is that of G.W.von Leibnitz (1646-1716). Leibnitz put forward a two-fold plan for the reform of logic. He suggested first the establishment of a universal scientific language in which all scientific concepts could be represented by a combination of basic ideograms. He suggested that a universal calculus of reasoning could be devised which would provide an automatic method of solution for all problems which could be expressed in the universal language. Had he carried out his proposal, he would have provided a system of symbolic logic.
The next important name in the development of symbolic logic is that of George Boole (1815-1864). His contribution consisted in the formulation of a system of algebra which was first set out in the book *The Mathematical Analysis of Logic*, and in a subsequent work *The Laws of Thought*. Boole applied his algebra to several branches of logic including the syllogism of the classical logic. This was an important advance in that he showed that the doctrine of the Aristotelian syllogism which had hitherto been regarded as practically co-extensive with deductive logic could be shown to be a special case of a kind of logical algebra.

Other important nineteenth century logicians who contributed to the development of symbolic logic included Augustus de Morgan (1806-1871), W.S. Jevons (1835-1882) and C.S. Peirce (1839-1914).

A number of mathematicians on the continent of Europe were interested themselves in the foundations of mathematics. Their works, in particular that of Gottlob Frege and Guiseppe Peano, was continued by Bertrand Russell. In 1910, in collaboration with A.N. Whitehead, Russell published *Principia Mathematica*, a monumental work in which a system of symbolic logic is elaborated and made to serve as the foundation of the whole of mathematics. The system of symbolic logic or mathematical logic set out by Russell and Whitehead embodied and consolidated the work of their predecessors and brought to the public notice the metamorphosis of logic which had taken place during the previous century. Since the publication of *Principia Mathematica*, logic has been a vigorously growing science. Thus the slow and largely unnoticed development of logic since the days of Leibnitz culminated in a work whose main object was mathematical. But symbolic logic is not important only for studies in the foundations of mathematics, though this is one field which it can be useful. It shares with the traditional logic the function of providing a method of testing the validity of the arguments of ordinary language and, it offers methods of deciding the validity of types of argument which cannot be tested by the classical logic. It provides a procedure for analyzing the structure of propositions. If symbolic logic is a developed form of the classical logic, it does all the tasks which the classical logic did and many others of which classical logic was not capable.

**LOGICAL FORM AND USES OF SYMBOLS**

**LOGICAL FORM IN SYMBOLIC LOGIC**

The form of a thing is distinguished from it’s matter. Form stands for the structure, while matter stands for the material of which it is made. For example, the form and matter of a table. The form of many tables may be the same (circular), but their matter may be different (wood, steel etc). Argument also has form and matter. The matter of an argument is the subject matter with which the argument deals. The structure or pattern of the argument is the form of the
argument. Just as the form or structure of a thing is determined by the way in which it’s parts are related, together, the form or structure of the argument is determined by the way in which it’s parts, the constituent propositions or terms (in the case of propositions) are related together.

In the following two arguments,

All men are mortal

Ramesh is a man

˙. Ramesh is mortal (1)

All students are honest

Dinesh is a student

˙. Dinesh is honest (2)

the matter is different, but the form is the same.

The two arguments can be expressed using symbols of Aristotelian logic as follows:

All M is P

S is M

˙. S is P (1)

All M is P

S is M

˙. S is p (1)

The two arguments,

If it rains, then the road will be wet

It rains

˙. The road will be wet (1)

If you study well, then you will pass the examination

You study well

˙. You will pass the examination (2)

have different matter, but the same form.
The form of the two arguments can be expressed using variables for propositions as follows:

If p, then q

\[ \begin{align*}
& \text{p} \\
& \therefore q \quad (1)
\end{align*} \]

If p then q

\[ \begin{align*}
& \text{p} \\
& \therefore q \quad (2)
\end{align*} \]

The structure, form, or organization of a thing is constituted by the way in which its parts are put together and by the mutual relations between them. We can speak of the logical form of a statement or the set of statements constituting an argument. By doing so, the form or structure of the statement or argument is distinguished from its subject matter. In logic, only the form of argument is important, not the subject matter. That is why in symbolic logic the words which refer to the subject matter is dispensed with and replaced by variables and constants. The use of symbols in logic and logical form are closely connected. In the classical logic, one of the main uses of a good symbolic notation is to show the logical form. One of the advantages of symbolic logic over its less developed classical logic is that it has a more complete symbolic device which enables to show the logical form of arguments than the Aristotelian logic. The reason for the logicians are interested in logical form is that logicians are interested in validity and the validity of arguments depends on their logical form. Two conditions are necessary to guarantee the truth of the conclusion of an argument. First, the evidence or premises from which deductions are made must be true. Secondly, the deductions must be valid. Of these two conditions, logic can guarantee only the second. The guarantee conferred by the validity of an argument is this:

If the premises of a valid argument are true, the conclusion is certainly true also. But where the premises are not true, the conclusion may be true or false even if the argument is valid. Thus logic does not concern directly with the factual truth of statements, even if those statements are premises or conclusions of arguments. Methods of testing the validity of various forms of arguments involve attention to the logical structure or logical form of arguments in that the validity of arguments is dependent on certain features of its logical form. To give attention to the logical form, it will be convenient to represent all arguments of a certain form by means of an appropriate symbolic notation in order that the relevant features of the structure be made clear.

**USE OF SYMBOLS IN SYMBOLIC LOGIC**

Arguments presented in English or any other natural language are often difficult to understand because of the vague and equivocal nature of the words used, the ambiguity of their construction, the misleading idioms they may
contain, their potentially confusing metaphorical style, and the distraction due to whatever emotive significance they may express. To avoid those peripheral difficulties, it is convenient to set up an artificial symbolic language, free from such defects, in which statements and arguments can be formulated. The use of special logical notation is not peculiar to modern logic.

Aristotle, the ancient founder of the subject, used variables to facilitate his own work. Although the difference in this respect between modern and classical logic is not one of kind but of degree, the difference in degree is tremendous. The greater extent to which modern logic has developed its own special technical language has made it immeasurably more powerful a tool for analysis and deduction. The special symbols of modern logic help us to show with greater clarity the logical structures of propositions and arguments whose forms may tend to be obscured by the unwieldiness of ordinary language.

1. The use of symbols in symbolic logic helps us to bring out the features of logical importance in arguments and classify them into types.

Logic provides methods of testing the validity of arguments. In order to do this, arguments are to be classified into different types such that each type of argument has certain features in common with others of the same type. The features which arguments have in common are called logical form of the argument. The traditional method of classifying arguments into types which was first invented by Aristotle involves the use of symbols. The use of variables in logic enables us to state general rules for testing the validity of arguments. Thus one important function of symbols in logic is to express the generality of the rules of logic.

2. Another important use of symbols in logic is to give conciseness and economy of expression to complicated statements which would be difficult or impossible to understand if they were expressed in ordinary language.

The situation here is comparable to the one leading to the replacement of Roman numerals by the Arabic notation. Arabic numerals are clearer and more easily comprehended than the older Roman numerals that they displaced.

3. Special symbols in logic representing logical operations facilitated the appraisal of arguments.

Technical terms in mathematics and logic are symbolized by special symbols. The drawing of inferences and the appraisal of arguments is greatly facilitated by the adoption of special logical notation. Alfred North Whitehead one of the great contributors to the advance of symbolic logic, stated “........by the aid of symbolism, we can make transitions in reasoning almost mechanically by the eye, which otherwise would call into play the higher faculties of the brain”.

Logic is not concerned with developing our powers of thought but with developing techniques that permit us to accomplish some tasks without having to think so much.
MODERN CLASSIFICATION OF PROPOSITIONS

All propositions are either simple or compound or general. A simple proposition is one which does not contain any other proposition as it’s component. A simple proposition cannot be divided into other propositions. A compound proposition is one which contains other proposition as it’s component. "Ramesh is honest" is a simple proposition. “Ramesh is honest and Dinesh is intelligent” is a compound proposition. It contains two simple propositions as it’s components. “Ramesh is not honest” is a compound proposition because it contains another proposition as it’s component, ‘Ramesh is honest’. Simple and compound propositions are also known as atomic and molecular propositions respectively. Compound propositions are of different types such as Negative, conjunctive, Disjunctive, Implicative, Material Equivalent.

Negation - Negation is a compound proposition in which the word ‘not” or the phrase ‘it is not the case that’ is used. Example,’Ramesh is not honest’ or ‘It is not the case that Ramesh is honest’. It contains ‘Ramesh is honest’ as component part.

Conjunction - Conjunction is a compound proposition in which the word “and” is used to connect simple statements. Example, ‘Ramesh is honest and Dinesh is intelligent’. It contains ‘Ramesh is honest’, ‘Dinesh is intelligent’ as components. The components of conjunction are called conjuncts.

Disjunction - Disjunction is a compound proposition in which the simple propositions are connected by the word ‘or ‘ or the phrase ‘either...........or’. Example, ‘Ramesh is either intelligent or hard working’. ‘Today is either Wednesday or Thursday’. The components of a disjunction are called disjuncts. Logicians recognize two kinds of disjunctions, inclusive disjunction and exclusive disjunction. A disjunction containing non-exclusive alternatives is called inclusive disjunction. Example, ‘Ramesh is either intelligent or hard working’. The sense of ‘or’ in inclusive disjunction is ‘at least one, both may be’. A disjunction containing exclusive alternatives is called exclusive disjunction. Example, ‘Today is either Wednesday or Thursday’. The sense of ‘or’ in exclusive disjunction is ‘at least one, not both’.

Implication - Implication is a compound proposition in which the simple statements are connected by the phrase ‘if ...... then’. For example, “If it rains, then the road will be wet”. The part of proposition which lies in between ‘if ‘ and ‘then’ is called antecedent. The part of proposition which follows the word ‘then’ is called consequent. The general form of an implicative proposition is as follows: “If antecedent, then consequent”.

Bi-conditional proposition: Bi-conditional proposition is a compound proposition in which the simple statements are connected by the phrase ‘if and only if’. For example, “I will go to the cinema if and only if my friend comes with me”. Bi-conditional proposition is also called ‘material equivalence’.

General proposition: A general proposition is a quantified statement. The categorical propositions of traditional logic are general propositions. There are
four types of general propositions, Universal Affirmative (A), Universal Negative (E), Particular Affirmative (I), Particular Negative (O).

Universal Affirmative (A): - Example, “All men are mortal”
Universal Negative: - Example, “No men are perfect”.
Particular Affirmative: - Example, “Some men are honest”
Particular Negative: - Example, “Some men are not honest”.

A general proposition may be simple or compound. The general proposition “All men are mortal” is a simple proposition. But the general proposition “All men are mortal and all human beings are lovable” is a compound proposition. Thus general propositions are either simple or compound depending on the nature of their components.

General propositions may be singly general or multiply general. Singly general proposition is a general proposition with one quantifier. Example, “All men are mortal”. Multiply general proposition is a general proposition with two or more quantifiers. Example, “All men are mortal and some men are honest”.

BRANCHES OF SYMBOLIC LOGIC –
PROPOSITIONAL LOGIC AND PREDICATE LOGIC

Symbolic logic contains two main branches, propositional logic and predicate logic. Statements in arguments may be simple, compound or general. The two main branches of symbolic logic namely propositional logic and predicate logic are classified according to the nature of propositions contained in arguments and the way in which they are symbolized according to the structure of statements.

Propositional logic is a branch of Symbolic Logic which deals with the validity of arguments containing simple statements and truth–functional compound statements. Since the validity of arguments are depending on the form of arguments, the structure of arguments are very important. In propositional logic, the form of arguments are represented using statement variables and logical constants. Simple statements are represented using statement variables. Compound statements in which simple statements are truth-functionally related are represented using statement variables and logical constants.

Predicate logic is a branch of symbolic logic which deals with the validity of arguments containing Singular and general propositions. General statements are represented using Predicate symbols, Individual constants and quantifiers, and logical constants wherever necessary. In predicate logic, the inner logical structure of statements are taken into consideration in representing the form of arguments. Individual constants and predicate symbols are used to represent subject and predicate respectively of statements.

A fundamental difference between the two branches of logic lies in analyzing the structure of propositions. Propositional logic does not analyse the
internal structure of propositions, whereas the predicate logic analyses the internal structure of propositions. A fundamental difference between the two branches of logic lies in analyzing the structure of propositions. Propositional logic does not analyse the internal structure of propositions, whereas the predicate logic analyses the internal structure of propositions. This may be illustrated with the help of a proposition such as ‘Ramesh is short’. It is symbolized with the help of a single letter, in propositional logic. It may be represented as ‘R’. In predicate logic the same proposition is symbolized as ‘Sr’, S stands for the predicate ‘short’ and ‘r’ stands for the subject ‘Ramesh’. In the second case the two terms constituting the proposition are taken into account.
UNIT II: PROPOSITIONAL LOGIC

SIMPLE AND COMPOUND PROPOSITIONS

Propositional Logic deals with arguments containing simple and compound statements. The two types of statements dealt within propositional logic are, simple and compound statements.

The modern logicians have broadly classified propositions into simple and compound propositions.

A simple statement is one that does not contain any other statement as a component. A simple proposition cannot be analysed into further propositions. For example, ‘Ramesh is honest’. A simple statement cannot be further analysed into statement or statements. ‘Ramesh is honest’ does not contain any other statement as a component.

A compound statement is one that does contain another statement as a component. For example, ‘Ramesh is not honest’. ‘Ramesh and Dinesh are honest’. A compound statement can be further analysed into a statement or statements. ‘Ramesh is not honest’ can be analysed into ‘Ramesh is honest’ which is a statement and ‘not’ which is a word. It contains ‘Ramesh is honest’ as a component. ‘Ramesh and Dinesh are honest’ can be further analysed into ‘Ramesh is honest’ which is a statement and ‘Dinesh is honest’ which is another statement. It contains ‘Ramesh is honest’ as a component and ‘Dinesh is honest’ as another component.

There are five types of compound propositions. They are, Negation, Conjunction, Disjunction, Conditional statement and Bi-Conditional statement.

TRUTH-FUNCTIONAL COMPOUND STATEMENTS:

Negation; Conjunction, Disjunction; Conditional Statement and Material Implication.

The notion of a function may be made clear with the help of an example from mathematics. Y=X+4 is an expression in mathematics. The value taken by X decides the value of Y. If X is 2, then Y must be 6. Here Y is a function of X, because the value of X decides the value of Y. In a similar way in the expression Z=3X-3Y+X, Z is said to be the function of X and Y. In logic, instead of the numerical values, truth-values namely true and false are used. The truth-value of a true proposition is “true” and the truth-value of a false proposition is “false”.

A compound proposition is truth-functional if and only if it's truth-value is completely determined by the truth-values of it's component statements. There are several types of truth-functional compound statements.

**Conjunction**: Conjunction is a compound proposition in which the word “and” is used to connect simple statements. Conjunction of two statements is formed by placing the word “and” between them. The two component statements of conjunction are called “conjuncts”. For example, ‘Ramesh is honest and Dinesh is intelligent’. ‘Ramesh is honest’ is the first conjunct and ‘Dinesh is intelligent’ is the second conjunct. To have a unique symbol whose only function is to connect statements conjunctively, the dot “.” symbol is used for conjunction. The above conjunction is represented as “Ramesh is honest. Dinesh is intelligent”. It is symbolized as” R.D” where’ R’ represents “Ramesh is honest, ‘D’ represents ‘Dinesh is intelligent”. More generally, where p and q are any two statements whatever their conjunction is symbolized as ‘p . q’.

In conjunction, if both it’s conjuncts are true , the conjunction is true, otherwise it is false. For this reason a conjunction is said to be a truth-functional compound statement. The dot “ .” symbol is a truth-functional connective.

Given any two statements, p and q, there are four possible sets of truth-values they can have, which can be displayed as follows:

- Where p is true and q is true, p.q is true.
- Where p is true and q is false, p.q is false.
- Where p is false and q is true, p.q is false.
- Where p is false and q is false, p.q is false.

If we represent the truth-values “true” and “false” by the capital letters T and F, the truth-table for conjunction can be represented as follows:

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p.q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

As shown by the truth-table defining the dot symbol, a conjunction is true if and only if both of it’s conjuncts are true.

**Negation**: Negation is a compound proposition in which the word ‘not” or the phrase ‘it is not the case that’ or the phrase “It is false hat”is used. Example,’Ramesh is not honest’or ‘It is not the case that Ramesh is honest’ or ‘It is false that Ramesh is honest’. It contains ‘Ramesh is honest’ as component part.
Negation of a statement is formed by the insertion of the word “not” in the original statement or by prefixing to it the phrase ‘it is not the case that’ or ‘it is false that’. The symbol “ ~” called “curl” or “tilde” is used to form the negation of a statement. Thus the above example is represented as ~R where R symbolizes ‘Ramesh is honest’. More generally, where p is any statement whatever, it’s negation is symbolized as ‘ ~ p’.

Negation is a **truth-functional compound statement** and the curl “ ~ ” is a **truth-functional operator**.

The negation of any true statement is false, and the negation of any false statement is true. The truth- table for Negation can be represented as follows:

<table>
<thead>
<tr>
<th>P</th>
<th>~P</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

**Disjunction:** Disjunction is a compound proposition in which the simple propositions are connected by the word ‘or ‘ or the phrase ‘either………..or’. Disjunction or alternation of two statements is formed by inserting the word “or” or the phrase “either….or” between them. The two component statements are called “disjuncts” or “alternatives”. For example, ‘A or B’, ‘Either A or B’.

There are two types of disjunction, Inclusive disjunction and Exclusive disjunction. The inclusive “or” has the sense of “at least one, both may be”. This sense of the word “or” is also called “weak”. For example, ‘Arun is either intelligent or honest’. The meaning is that Arun is intelligent only, or honest only, or both intelligent and honest. The exclusive “or” has the sense of “at least one, not both”. This sense of the word “or” is also called “strong”. For example, ‘Today is Sunday or Monday’. The meaning is that today is Sunday or Monday but not both Sunday and Monday. The two kinds of disjunction have a part of their meanings in common. This partial common meaning, that ‘at least one’ is the whole meaning of the inclusive “or” and part of the meaning of the exclusive “or”. The common meaning, that is, the inclusive sense is represented by the Latin word “ vel”. The initial letter of the word “vel” is used to represent the inclusive sense. Where p and q are any two statements whatever, their inclusive disjunction is written as “ p v q”. Inclusive or weak disjunction is false only in case both of it’s disjuncts are false. The symbol “v” is a truth-functional connective. The four truth-value possibilities are

where “p” is true and “q” is true, “p v q” is true.

Where” p” is true and “q” is false,”p v q” is true.

Where “p” is false and “q” is true, “p v q” is true.

Where “p” is false and “q” is false, “p v q” is false.
The truth-table for disjunction is as follows:

\[
\begin{array}{ccc}
\text{P} & \text{q} & \text{pvq} \\
T & T & T \\
T & F & T \\
F & T & T \\
F & F & F \\
\end{array}
\]

The word “unless” is used to form the disjunction of two statements. Thus the statement “You will fail in the exam unless you study well” is symbolized as “FvS”.

**Implication or conditional statement:** Implication is a compound proposition in which the simple statements are connected by the phrase ‘if …… then’. For example, “If it rains, then the road will be wet”. The part of proposition which lies in between ‘if’ and ‘then’ is called the **antecedent** or **implicans**. The part of proposition which follows the word ‘then’ is called the **consequent or implicate**. The general form of an implicative proposition is as follows: “If **antecedent**, then **consequent**”. When two statements are combined by placing the word “if” before the first and inserting the word “then” between them, the resulting compound statement is called conditional, hypothetical, implication or implicative statement.” If….then” is the phrase used in conditional statement. For example, ‘If it rains, then the road will be wet.’ It rains’ is the antecedent, and ‘The road will be wet’ is the consequent.

A conditional statement asserts that in any case in which it's antecedent is true, it’s consequent is true also. It does not assert that it's antecedent is true, but only that if it’s antecedent is true, it’s consequent is true also.

There are different types of implication each of which asserts a different sense of “if…..then”.

In the statement ‘If all humans are mortal and Socrates is a human, then Socrates is mortal’, the consequent follows logically from it’s antecedent.

In the statement ‘If Ramesh is a bachelor, then Ramesh is unmarried’, the consequent follows from it’s antecedent by the very definition of the term ‘bachelor’, which means unmarried man.

In the statement, ‘If blue litmus paper is placed in acid, then it will turn red’, the relation between antecedent and consequent is causal. In the example, ‘If Indian cricket team loses it’s match against Australia, then I will eat my hat’, the decision of the speaker is reported.

The above four types of conditional statements are all conditional statements. The common meaning, that is part of the meaning of all four different types of implication, is to be found out. Conditional statement does not assert that it’s antecedent is true, but asserts that if the antecedent is true, then it’s consequent is also true. Conditional statement is false if it’s antecedent is
true and consequent is false. That is, it cannot be the case that the antecedent is true and consequent false. Any conditional statement “If $p$ then $q$” in case the conjunction “$p \land \neg q$" is known to be is false. For a conditional to be true the conjunction “$p \land \neg q$“ must be false. That is, it’s negation $\neg (p \land \neg q)$ must be true. Thus, $\neg (p \land \neg q)$ is regarded the common meaning that is part of the meaning of all four different types of implication symbolized as “If $p$, then $q$”. Every conditional statement means to deny that it’s antecedent is true and it’s consequent false. This common partial meaning of the “if....then” is symbolized as “$\supset$” called “horseshoe”. “If $p$ then $q$” is symbolized as $p \supset q$ as an abbreviation for $\neg (p \land \neg q)$.

The symbol “$\supset$” can be regarded as representing another kind of implication called “material implication”. No real connection between antecedent and consequent is suggested by material implication. It asserts that “it is not the case that the antecedent is true and the consequent is false”. All types of implication can be treated as material implication. The material implication (implication) symbol is a truth-functional connective. The possible combinations of truth-values for implication is represented as follows:

- Where $p$ is true and $q$ is true, $p \supset q$ is true.
- Where $p$ is true and $q$ is false, $p \supset q$ is false.
- Where $p$ is false and $q$ is true, $p \supset q$ is true.
- Where $p$ is false and $q$ is false, $p \supset q$ is true.

The truth-table for implication is as follows:

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \supset q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

If it rains, then the road will be wet
If it rains, the road will be wet
The road will be wet if it rains
It rains implies that the road will be wet.
The road being wet is implied by it rains.

All these conditional statements are symbolized as $R \supset W$

Where $p$ and $q$ are any two statements implication between any two statements is symbolized as “$p \supset q$“. This can also be read as “$p$ is a sufficient condition for $q$”, “$q$ is a necessary condition for $p$”, “$p$ only if $q$”.

Bi-conditional proposition: Bi-conditional proposition is a compound proposition in which the simple statements are connected by the phrase ‘if and only if’. For example, “I will go to the cinema if and only if my friend comes with me”. Bi-conditional proposition is also called ‘material equivalence’. When two statements are combined by using the phrase “if and only if”, the resulting compound statement is called bi-conditional statement or Material Equivalence. The phrase “if...then” is used to express sufficient condition, “only...if” phrase is used to express necessary condition. The phrase “if and only if” is used to express both sufficient and necessary condition. The above example can also be stated as, ‘I will go to the cinema if my friend comes with me and, I will go to the cinema only if my friend comes with me’. The phrase “If and only if” is symbolized as ’≡ ‘.

Where \( p \) and \( q \) are any two statements whatever, their material equivalence is represented as ‘ \( P \equiv q \)’. When two statements are materially equivalent, they materially imply each other.

The possible combinations of truth-values for material equivalence is as follows:

- Where \( p \) is true and \( q \) is true, \( p \equiv q \) is true.
- Where \( p \) is true and \( q \) is false, \( p \equiv q \) is false.
- Where \( p \) is false and \( q \) is true, \( p \equiv q \) is false.
- Where \( p \) is false and \( q \) is false, \( p \equiv q \) is true.

Two statements are said to be materially equivalent when they have the same truth-value. The symbol ’≡ ‘ is a truth-functional connective. The truth-table for biconditional or material equivalence is as follows:

<table>
<thead>
<tr>
<th></th>
<th>( p \equiv q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

PUNCTUATION IN SYMBOLIC LOGIC

In English, punctuation is absolutely required if complicated statements are to be clear. Punctuation is equally necessary in mathematics. Punctuation marks in mathematics appear in the form of parentheses, brackets and braces. In symbolic logic, those same punctuation marks are equally essential, because in logic compound statements are themselves compounded together into more complicated ones. Thus \( p \cdot (q \lor r) \) is ambiguous. It might mean the conjunction of \( p \) with the disjunction of \( q \) with \( r \), or it might mean the disjunction whose first disjunct is the conjunction of \( p \) and \( q \) and whose second disjunct is \( r \). These two different senses are distinguished by punctuating the given formula as \( p \cdot (q \lor r) \) or else as \( (p \cdot q) \lor r \).
The negation of a disjunction is formed by the phrase “neither-nor”. The negation of $F \lor H$ is symbolized as $\neg (F \lor H)$ or as $(\neg F) \cdot (\neg H)$.

The order of the words “both” and “not” is very important. There is a difference between

Ramesh and Dinesh will not both be elected.

and

Ramesh and Dinesh will both not be elected.

The first denies the conjunction $R \cdot D$ and is symbolized as $\neg (R \cdot D)$. The second says that each one will not be elected and is symbolized as $(\neg R) \cdot (\neg D)$.

In any formula, the negation symbol will be understood to apply to the smallest statement that the punctuation permits. Without this convention $\neg p \lor q$ is ambiguous, meaning either $(\neg p) \lor q$ or $\neg (p \lor q)$. By our convention it means the first, $(\neg p) \lor q$.

The exclusive disjunction of $p$ and $q$ asserts that at least one of them is true but not both are true which is written as $(p \lor q) \cdot \neg (p \cdot q)$.

Thus brackets are used as punctuation marks in symbolic logic. The use of brackets decides the scope of constants.

ARGUMENT FORMS AND ARGUMENTS

Refutation by logical analogy:- One way of proving invalidity of arguments is by the method of logical analogy. An argument can be proved invalid by constructing another argument of the same form with true premises and false conclusion. An argument with true premises and false conclusion cannot be valid, but can only be invalid. To prove the invalidity of an argument, it is sufficient to construct another argument of the same form with true premises and false conclusion. This method is based upon the fact that validity and invalidity are purely formal characteristics of arguments. That is to say, any two arguments having the same form are either both valid or both invalid, regardless of the subject matter of the arguments.

The form of arguments are represented by means of statement variables. A statement variable is a letter for which or in place of which a statement may be substituted. Small letters from the middle part of the alphabet $p, q, r, s$ ....are used as statement variables. Compound statements as well as simple statements may be substituted for statement variables.

An argument form can be defined as an array of symbols containing statement variables but no statements, such that when statements are substituted for statement variables-the same statement being substituted for the same statement variable throughout – the result is an argument.
The two arguments,
\[ A \supset B \quad \text{and} \quad M \supset N \]

have the same form which is represented as
\[ p \supset q \]
\[ q \]
\[ \therefore p \]

Any argument that results from the substitution of statements for statement variables in an argument form is called a “substitution instance” of that argument form.

For any argument, there are several argument forms that have the given argument as a substitution instance. For example,
\[ A \supset B \]
\[ B \]
\[ \therefore A \]

Is a substitution instance of the argument forms
\[ p \supset q \quad \text{and} \quad p \]
\[ q \]
\[ \therefore p \]
\[ q \]
\[ \therefore r \]
The first argument form is called the “specific form” of the given argument.

In case an argument is produced by substituting a different simple statement for each different statement variable in an argument form, that argument form is called the “specific form” of that argument.

If the specific form of a given argument has any substitution instance whose premises are true and whose conclusion is false, then the given argument is invalid.

An argument form is invalid if and only if it has at least one substitution instance with true premises and false conclusion.

Refutation by logical analogy is based on the fact that any argument whose specific form is an invalid argument form is an invalid argument.

Any argument form that is not invalid must be valid. An argument form is valid if and only if it has no substitution instances with true premises and false
conclusion. An argument is valid if and only if the specific form of that argument is a valid argument form.

A given argument is proved invalid if a refuting analogy for it can be constructed. Given any argument, the specific form of the argument is tested, for it’s validity or invalidity determines the validity or invalidity of the argument.

To test an argument form, all possible substitution instances of it is examined to see if any one of them has true premises and false conclusion. All possible substitution instances whose premises and conclusions have different truth-values is obtained by examining all possible different arrangements of truth-values for the statements that can be substituted for the different statement variables in the argument form to be tested. Where an argument form contains two different statement variables, p and q, all of it’s substitution instances are assembled most conveniently in the form of a truth-table. To decide the validity of the argument form

\[ P \supset q \]
\[ q \]
\[ \therefore p \]

The following truth-table is constructed:

<table>
<thead>
<tr>
<th>P</th>
<th>q</th>
<th>P \supset q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

Each row of this table represents a whole class of substitution instances. The third column heading is the first premise, the second column is the second premise, and the first column is the conclusion. In examining this truth-table, in the third row there are T’s under both premises and an F under the conclusion, which indicates that there is at least one substitution instance of this argument form that has true premises and a false conclusion. This row suffices to show that the argument form is invalid.

Valid argument forms:-

**Disjunctive Syllogism:**

\[ P \lor q \]
\[ \sim p \]
\[ \therefore q \]

To show the validity, the following truth-table is constructed:
The third row is the only one in which T’s appear under both premises and T appear under the conclusion also. The truth-table shows that the argument form has no substitution instance having true premises and a false conclusion and thereby proves the validity of the argument form.

**Modus Ponens:**

\[
P \rightarrow q \\
\neg q \\
\therefore \neg P
\]

The validity of Modus Ponens is proved by the following truth-table:

<table>
<thead>
<tr>
<th>P</th>
<th>q</th>
<th>P \rightarrow q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

Only the first row represents substitution instances in which both premises are true, and the T in the second column shows that the conclusion is true also.

**Modus Tollens:**

\[
P \rightarrow q \\
\neg q \\
\therefore \neg P
\]

The validity of Modus Tollens is shown by the following truth-table:

<table>
<thead>
<tr>
<th>P</th>
<th>q</th>
<th>P \rightarrow q</th>
<th>\neg q</th>
<th>\neg P</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

There is no substitution instances in which the premises are true and the conclusion false.
Invalid Argument Forms:

Fallacy of Affirming the Consequent-

\[ P \supset q \]

\[ q \]

\[ \therefore P \]

Fallacy of denying the antecedent-

\[ P \supset q \]

\[ \sim P \]

\[ \therefore \sim q \]

STATEMENT FORMS AND STATEMENTS:-

A statement form is any sequence of symbols containing statement variables but no statements, such that when statements are substituted for the statement variables-the same statement being substituted for the same statement variable throughout- the result is a statement. Thus, \( p \lor q \) is a statement form called “disjunctive statement form”. \( P \cdot q \) is conjunctive statement form. \( P \supset q \) is conditional statement form. \( \sim p \) is called a negation form or denial form. Any statement of a certain form is said to be a substitution instance of that statement form.

The specific form of a given statement is defined as that statement form from which the statement results by substituting a different simple statement for each different statement variable. For example, \( p \lor q \) is the specific form of the statement \( A \lor B \).

Tautologous, Contradictory, and Contingent statement forms:

The statement “Lincoln was assassinated”, symbolized as \( L \), is true as a historical fact. The statement “Washington was assassinated” symbolized as \( W \), is false as a historical fact. But the statement \( L \lor \sim L \) is true in virtue of it’s form alone, a logical truth, a formal truth. It is a substitution instance of a statement form all of whose substitution instances are true statements. Similarly the statement \( W \lor \sim W \) is false in virtue of it’s form alone, a logical truth, a formal truth. It is a substitution instance of a statement form all of whose substitution instances are false.

A statement form that has only true substitution instances is called a “tautologous statement form” or a “tautology”. The statement form \( p \lor \sim p \) is a tautology. The truth-table for \( p \lor \sim p \) is represented as follows:
Any statement that is a substitution instance of a tautologous statement form is said to be tautologous or a tautology.

A statement form that has only false substitution instances is said to be “self-contradictory” or a “contradiction”. The statement form p . ~ p is self-contradictory. The truth-table for p . ~ p is represented as follows:

<table>
<thead>
<tr>
<th>p</th>
<th>~p</th>
<th>p v ~p</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

Any statement that is a substitution instance of self-contradictory statement form is said to be self-contradictory or contradiction.

Statement forms that have both true and false statements among their substitution instances are called “contingent statement forms”.

Any statement whose specific form is contingent is called a “contingent statement”. Thus p, ~ p, p . q, p v q and p \(\Leftrightarrow\) q are all contingent statement forms.

**Logical Equivalence :**

Two statements are logically equivalent when their material equivalence is a tautology. Thus the “principle of double negation”, p \(\equiv\) ~ ~ p is proved to be tautologous by the following truth-table:

<table>
<thead>
<tr>
<th>p</th>
<th>~p</th>
<th>~ ~ p</th>
<th>p (\equiv) ~ ~ p</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

Which proves the logical equivalence of p \(\equiv\) ~ ~ p

There is difference between logical equivalence and material equivalence. Two statements are logically equivalent only when it is absolutely impossible for the two statements to have different truth-values. Therefore, logically equivalent statements have the same meaning and may be substituted for one another. But two statements are materially equivalent if they merely happen to have the same truth-value. Statements that are merely materially equivalent certainly may not be substituted for one another.

**De Morgan’s Theorems:**

There are two logical equivalences that express the interrelations among conjunction, disjunction and negation. The disjunction p v q asserts that at least one of it’s two disjuncts is true, it is contradicted only by asserting that both are false. Thus asserting the negation of the disjunction p v q is logically
equivalent to asserting the conjunction of the negations of $p$ and of $q$. This is expressed by the biconditional $\sim (p \lor q) \equiv (\sim p \cdot \sim q)$.

Similarly, asserting the conjunction of $p$ and $q$ asserts that both are true. To contradict $p \cdot q$ is to assert that at least one is false. Thus asserting the negation of the conjunction $p \cdot q$ is logically equivalent to asserting the disjunction of the negations of $p$ and of $q$. This is expressed by the biconditional $\sim (p \cdot q) \equiv (\sim p \lor \sim q)$ which is proved to be a tautology. The above two biconditionals are known as De Morgan’s theorems, having been stated by the mathematician and logician Augustus De Morgan. De Morgan’s theorems is formulated as:

\[
\text{Disjunction} \iff \text{conjunction} \\
\text{The negation of the conjunction of two statements is logically equivalent to the disjunction of the negations of the two statements.}
\]

Arguments, conditional statements, and tautologies:-

To every argument there corresponds a conditional statement whose antecedent is the conjunction of the argument’s premises, and whose consequent is the argument’s conclusion. Thus the argument,

\[
P \supset q \\
P \\
\therefore q
\]

may be expressed as a conditional statement of the form $[(P \supset q) \cdot p] \supset q$.

An argument form is valid if and only if it’s truth-table has a T under the conclusion in every row in which there are T’s under all of its premises. Only T’s will occur under a conditional statement that correspond to a valid argument, and that conditional statement must be a tautology.

An argument form is valid if and only if it’s expression in the form of a conditional statement is a tautology.

For every invalid argument the corresponding conditional statement will not be a tautology.

The paradoxes of material implication:- The statement forms, $p \supset (q \supset p)$ and $\sim p \supset (P \supset q)$ are proved to be tautologies. The first may be stated as “If a statement is true, then it is implied by any statement whatever”. The second may be stated as “If a statement is false, then it implies any statement whatever”. These express the meaning of material implication. These statement forms when expressed in language seem to be paradoxical. But subject matter or meaning is strictly irrelevant to material implication. Only truth and falsehood are relevant here.
METHOD OF DEDUCTION –
FORMAL PROOF OF VALIDITY

Truth-table method becomes unwieldy as the number of component statements increases. A more efficient method of establishing the validity of an extended argument is to deduce it’s conclusion from it’s premises by a sequence of elementary arguments each of which is known to be valid.

The argument,

\[
\begin{align*}
& A \supset B \\
& B \supset C \\
& C \supset D \\
& \sim D \\
& A \lor E \\
& \therefore E
\end{align*}
\]

requires 32 rows in the truth-table method. But the argument can be proved valid by deducing its conclusion from its premises by a sequence of four elementary valid arguments. From \( A \supset B \) and \( B \supset C \), \( A \supset C \) is inferred by Hypothetical Syllogism. From \( A \supset C \) and \( C \supset D \), \( A \supset D \) is inferred by Hypothetical Syllogism. From \( A \supset D \) and \( \sim D \), \( \sim A \) is inferred by modus tollens. From \( \sim A \) and \( A \lor E \), \( E \), the conclusion of the original argument is inferred by Disjunctive Syllogism.

That the conclusion can be deduced from the premises by four elementary valid arguments proves the original argument to be valid. The elementary valid argument forms, hypothetical syllogism, modus tollens, and disjunctive syllogism are used as rules of inference.

The formal proof of the argument is written as,

1. \( A \supset B \)
2. \( B \supset C \)
3. \( C \supset D \)
4. \( \sim D \)
5. \( A \lor E \) / \( \therefore E \)
6. \( A \supset C \quad 1,2, \text{H.S} \)
7. \( A \supset D \quad 6,3, \text{H.S} \)
8. \( \sim A \quad 7,4, \text{M.T} \)
9. \( E \quad 5,8, \text{D.S} \)
A formal proof of validity for a given argument is defined as a sequence of statements each of which is either a premise of that argument or follows from preceding statements by an elementary valid argument such that the last statement in the sequence is the conclusion of the argument whose validity is being proved.

An elementary valid argument is defined as any argument that is a substitution instance of an elementary valid argument form. There are nine elementary valid argument forms accepted as Rules of Inference. They are:

1. Modus Ponens (M.P)
   \[ P \supset q \]
   \[ P \]
   \[ \therefore q \]

2. Modus Tollens (M.T)
   \[ P \supset q \]
   \[ \sim q \]
   \[ \therefore \sim p \]

3. Hypothetical Syllogism (H.S)
   \[ P \supset q \]
   \[ q \supset r \]
   \[ \therefore p \supset r \]

4. Disjunctive Syllogism (D.S)
   \[ p \lor q \]
   \[ \sim p \]
   \[ \therefore q \]

5. Constructive Dilemma (C.D)
   \[ ( P \supset q ) \cdot ( r \supset s ) \]
   \[ P \lor r \]
   \[ \therefore q \lor s \]

6. Absorption (Abs)
   \[ P \supset q \]
   \[ \therefore P \supset ( p \cdot q ) \]

7. Simplification (Simp)
   \[ P \cdot q \]
   \[ \therefore p \]
8. Conjunction (Conj)
   \[ P \]
   \[ Q \]
   \[ . \cdot . p \cdot q \]

9. Addition (Add)
   \[ P \]
   \[ . \cdot . p v q \]

THE RULE OF REPLACEMENT

There are many valid truth-functional arguments that cannot be proved valid using only the nine Rules of Inference. For example, a formal proof of validity for the valid argument

\[ A \cdot B \]
\[ . \cdot . B \]

requires additional Rules of Inference. In any truth-functional compound statement, if a component statement in it is replaced by another statement having the same truth-value, the truth-value of the compound statement will remain unchanged. The Rule of Replacement is an additional principle of inference. The Rule of Replacement permits to infer from any statement the result of replacing any component of that statement by any other statement logically equivalent to the component replaced.

For example, using the principle of Double Negation, which asserts that \( p \) is logically equivalent to \( \sim \sim p \), from \( A \supset \sim \sim B \) any of the following can be inferred by the Rule of Replacement.

\[ A \supset B, \sim \sim A \supset \sim \sim B, \sim \sim ( A \supset \sim \sim B), \text{ or } A \supset \sim \sim \sim \sim B \]

Tautologous or logically true biconditionals provide additional rules of inference in proving the validity of arguments. Any of the following logically equivalent expressions may replace each other wherever they occur.:

10. De Morgan’s Theorems (De M)-
    \[ ( P \cdot Q) \equiv (\sim P V \sim Q) \]
    \[ \sim ( P V Q) \equiv (\sim P . \sim Q) \]

11. Commutation (Com)-
    \[ ( p v q) \equiv (q v p) \]
    \[ (p . q) \equiv (q . p) \]

12. Association (Assoc)-
    \[ [ p v (q v r)] \equiv [ (p v q) v r ] \]
    \[ [ p . (q . r)] \equiv [ (p . q) . r ] \]

13. Distribution (Dist)-
    \[ [ p . (q v r)] \equiv [ (p . q) v (p . r) ] \]
    \[ [ p v (q . r)] \equiv [ (p v q) . (p v r) ] \]
14. Double Negation (D.N.) -
   \[ P \equiv \sim \sim p \]

15. Transposition (Trans) -
   \[ (P \supset q) \equiv (\sim q \supset \sim p) \]

16. Material Implication (Impl) -
   \[ (P \supset q) \equiv (\sim P \lor q) \]

17. Material Equivalence (Equiv) -
   \[ (P \equiv q) \equiv [(P \supset q) \cdot (q \supset p)] \]

   \[ (P \equiv q) \equiv [(p \cdot q) \lor \sim (P \cdot \sim q)] \]

18. Exportation (Exp) -
   \[ [(P \cdot Q) \supset r] \equiv [p \supset (q \supset r)] \]

19. Tautology (Taut) -
   \[ p \equiv (p \lor p) \]
   \[ p \equiv (p \cdot p) \]

Replacement is different from substitution. Statements are substituted for statement variables, whereas statements are replaced by other statements. We can substitute any statement for any statement variable, provided that if a statement is substituted for one occurrence of a statement variable it must be substituted for every other occurrence of that statement variable. But in moving from one statement to another by way of replacement, we can replace a component of the first only by a statement logically equivalent to that component, and we can replace one occurrence of that component without having to replace any other occurrence of it.

There is an important difference between the first nine and the last ten rules of inference. The first nine rules can be applied only to whole lines of a proof. On the other hand, any of the last ten rules can be applied either to whole lines or to parts of lines.

**PROVING INVALIDITY**

The invalidity of an argument can be proved by a truth-table to show that the specific form of that argument is invalid. The truth-table proves invalidity if it contains at least one row in which truth values are assigned to the statement variables in such a way that the premises are made true and the conclusion false. Truth value assignment without constructing the entire truth-table is a shorter method of proving invalidity. The argument,

\[
\begin{align*}
V \supset O \\
H \supset O \\
\therefore V \supset H
\end{align*}
\]

can be proved invalid by making an assignment of truth-values to the component simple statements V, O and H which will make the premises true and the conclusion false. The conclusion is made false by assigning true truth value to V and false truth value to H, and both premises are made true by assigning true truth value to O.

The truth value assignments are written horizontally as follows:
Making the indicated truth value assignment amounts to describing one row of the entire truth-table.

This method of proving invalidity is shorter than writing out a complete truth-table. This method will suffice to prove the invalidity of any argument which can be shown to be invalid by a truth-table.
UNIT III

PREDICATE LOGIC

THE NEED OF PREDICATE LOGIC:

The logical methods of distinguishing between valid and invalid arguments whose validity depends only upon the ways in which simple statements are truth-functionally combined into compound statements cannot be applied to arguments containing singular and general statements. There are arguments whose validity rests on the internal structure of propositions, and since the propositional logic does not deal with the internal structure of propositions, it cannot establish the validity of such arguments. Only the predicate logic which is concerned with the internal structure of propositions can properly handle those arguments whose validity goes with the internal structure of propositions.

All humans are mortal

Socrates is human

Therefore, Socrates is mortal.

The above valid argument, in propositional logic may be symbolized as,

\[ H \Rightarrow M \]

The invalidity of this argument can be proved by assigning truth values in such a way that premises are true and conclusion false.

The validity of this argument rests with the internal structure of the constituent propositions. Predicate logic is concerned with the internal structure of propositions. The statement “Ramesh is tall” is symbolized as \( R \) in propositional logic. In predicate logic it is symbolized as \( T_k \) where \( T \) stands for the predicate “is tall” and \( k \) stands for the particular individual Krishna. The method of validity in predicate logic is different from that of propositional logic. This justifies the need of predicate logic.

QUANTIFICATION THEORY

SINGULAR AND GENERAL PROPOSITIONS: QUANTIFICATION

“Socrates is human” is the simplest kind of noncompound proposition. It is called ‘singular proposition’.

An affirmative singular proposition asserts that a particular individual has a specified attribute. In the given example, ‘Socrates’ is the subject term and ‘human’ is the predicate term. The subject term denotes a particular individual and the predicate term designates some attribute the individual is said to have.
In symbolizing singular propositions small letters ‘a’ through ‘w’ are used to denote individuals. Since these symbols denote individuals, they are called ‘individual constants’. To designate attributes, capital letters are used. In the example, the small letter ‘s’ is used to denote Socrates and the capital letter ‘H’ is used to symbolize the attribute ‘human’. To express a singular proposition in symbolism, the symbol for predicate term is written to the left of the symbol for subject term. Thus, ‘Socrates is human’ is symbolized as ‘Hs’.

Symbolic formulations of Singular propositions having the same predicate term have a common pattern. For example, ‘Ha’, ‘Hb’, ‘Hc’, ‘Hd’ ………., the attribute symbol ‘H’ followed by an individual constant. The expression ‘Hx’ is used to symbolize the common pattern. The small letter ‘x’ called an ‘individual variable’ is a mere place marker that serves to indicate where an individual constant can be written to produce a singular proposition. ‘Hx’ is neither true nor false. Such expressions as ‘Hx’ are called ‘propositional functions’. Propositional functions are defined as expressions which contain individual variables and become propositions when their individual variables are replaced by individual constants. Any singular proposition can be regarded as a substitution instance of the propositional function from which it results by the substitution of an individual constant for the individual variable in the propositional function. The process of obtaining a proposition from a propositional function by substituting a constant for a variable is called instantiation. The negative singular proposition ‘Aristotle is not human’ symbolized as ‘ ~ Ha ’ result by instantiation from the propositional function ‘~ Hx ’, of which it is a substitution instance.

General propositions such as ‘Everything is mortal’, ‘Something is mortal’ differ from singular propositions in not containing the names of any individuals. They can be regarded as resulting from propositional functions by a process called ‘generalization ’ or ‘quantification’. The proposition ‘Everything is mortal’ can be expressed as, Given any individual thing whatever, it is mortal.

The relative pronoun ‘it’ refers back to the word ‘thing’ in the statement. By using the individual variable ‘x’ in place of ‘it and the word ‘thing’, the general proposition can be written as, Given any x , x is mortal.

By using propositional function, it can be re-written as,

Given any x, Mx

The phrase ‘Given any x’ is called a ‘universal quantifier’ and is symbolized as ‘(x)’. The general proposition can be completely symbolized as, (x)Mx

The proposition ‘ something is mortal’ can be written as follows:

There is at least one thing which is mortal.
There is at least one thing such that it is mortal.
There is at least one x such that  x is mortal.
There is at least one x such that  Mx
The phrase ‘there is at least one x such that’ is called an ‘existential quantifier’ and is symbolized as, ‘(∃x)’. The general proposition can be completely symbolized as, (∃X Mx).

A general proposition is formed from a propositional function by placing either a universal or an existential quantifier before it. The universal quantification of a propositional function is true if and only if all of it’s substitution instances are true. The existential quantification of a propositional function is true if and only if it has at least one true substitution instance. If we grant that there is at least one individual then every propositional function has at least one propositional function (true or false). The other two general propositions, ‘Something is not mortal’ and ‘Nothing is mortal’ are the respective negations of the first two general propositions.

‘Something is not mortal’ is symbolized as,

(∃x) ~ Mx

‘Nothing is mortal’ is symbolized as,

(x) ~ Mx

The negation of the universal(existential) quantification of a propositional function is logically equivalent to the existential(universal) quantification of the new propositional function which results from placing a negation symbol in front of the first propositional function. Where the Greek letter phi represent any attribute symbol whatever, the general connections between universal and existential quantification can be represented in terms of the following square array:

(x) φx  Contraries  (x) ~ φx

(∃x) φx  Sub contraries  (∃x) ~ φx

Assuming the existence of at least one individual, the two top propositions are contraries; they might both be false but cannot both be true. The two bottom propositions are subcontraries; they can both be true but cannot both be false. Propositions which are at opposite ends of the diagonals are contradictories, of which one must be true and the other false. On each side, the truth of the lower proposition is implied by the truth of the proposition which is directly above it.

**Symbolization of Traditional Subject – Predicate Propositions.**

Traditional logic emphasized four types of subject-predicate propositions illustrated by the following:
All humans are mortal.
No humans are mortal
Some humans are mortal
Some humans are not mortal

These were classified as ‘Universal affirmative’, ‘universal negative’, ‘particular affirmative’, and ‘particular negative’ respectively, and their types abbreviated as ‘A’, ‘E’, ‘I’, ‘O’ respectively. These four traditional subject-predicate propositions are symbolized by means of propositional functions and quantifiers.

The A proposition ‘All men are mortal’ can be re-written as,

Given any individual thing whatever, if it is human then it is mortal.
Given any x, if x is human then x is mortal.
Given any x, x is human → x is mortal

and finally symbolized as,

\((x) (H \land Mx)\)

The E proposition ‘No humans are mortal’ may be re-written as,

Given any individual thing whatever, if it is human then it is not mortal.
Given any x, if x is human then x is not mortal.

and finally symbolized as,

\((x) (H \rightarrow \neg Mx)\)

The I ‘proposition’ ‘Some humans are mortal’ may be re-written as,

There is at least one thing which is human and mortal
There is at least one thing such that it is human and it is mortal.
There is at least one x such that x is human and x is mortal.

and completely symbolized as,

\((\exists x) (H \land Mx)\)

The O ‘proposition’ ‘Some humans are not mortal’ is re-written as follows:

There is at least one thing which is human but not mortal.
There is at least one thing such that it is human and it is not mortal.
There is at least one \( x \) such that \( x \) is human and \( x \) is not mortal.

and completely symbolized as the existential quantification of a complex propositional function

\[
(\exists x) \ (H x \cdot M x)
\]

Where the Greek letters \( \phi \) and \( \psi \) are used to represent any attribute symbols whatever, the four general subject-predicate propositions of traditional logic may be represented in a square array as

\[
\begin{array}{cc}
(x)(\phi x \land \psi x) & (x)(\phi x \land \lnot \psi x) \\
(\exists x) (\phi x \land \psi x) & (\exists x) (\phi x \land \lnot \psi x)
\end{array}
\]

Of these, the \( A \) and the \( O \) are contradictories, \( E \) and \( I \) are contradictories also. But none of the other relationships hold even where we assume that there is at least one individual in the universe. \( A \) and \( E \) propositions are not contraries. \( I \) and \( O \) propositions are not sub contraries. The truth of universal does not imply the truth of particular also.

There are other general propositions that involve the quantification of more complicated propositional functions. Thus the general proposition ‘All members are either parents or teachers’ is symbolized as \( (x) \ [M x \lor (P x \lor T x)] \). The general proposition ‘Some Senators are either disloyal or misguided’, is symbolized as \( (\exists x) [S x \cdot (D x \lor M x)] \). The proposition ‘Apples and bananas are nourishing’ can be symbolized either as \( (x) \ [A x \lor B x] \lor (A x \land B x) \) or as

\[
\{ (x) \ [A x \lor B x] \} \lor \{ (x) \ [B x \lor A x] \}.
\]
**QUESTIONS**

**A-Short Answer Questions (1 weightage)**

1. Define conjunction

   Conjunction is a compound proposition in which the word “and” is used to connect simple statements. Example, ‘Ramesh is honest and Dinesh is intelligent’. It contains ‘Ramesh is honest’, ‘Dinesh is intelligent’ as components. The components of conjunction are called conjuncts.

2. Define negation

   Negation is a compound proposition in which the word ‘not” or the phrase ‘it is not the case that’ is used. Example, ‘Ramesh is not honest’ or ‘It is not the case that Ramesh is honest’. It contains ‘Ramesh is honest’ as component part.

3. Define disjunction

   Disjunction is a compound proposition in which the simple propositions are connected by the word ‘or’ or ‘or the phrase ‘either……….or’. Example, ‘Ramesh is either intelligent or hard working”. ‘Today is either Wednesday or Thursday’. The components of a disjunction are called disjuncts.

4. Define implication

   Implication is a compound proposition in which the simple statements are connected by the phrase ‘if …… then’. For example, “If it rains, then the road will be wet”. The part of proposition which lies in between ‘if ‘ and ‘then’ is called antecedent. The part of proposition which follows the word ‘then’ is called consequent.

5. Define bi-conditional

   Bi-conditional proposition is a compound proposition in which the simple statements are connected by the phrase ‘if and only if’. For example, “I will go to the cinema if and only if my friend comes with me”. Bi-conditional proposition is also called ‘material equivalence’.

6. Define compound statement

   A compound proposition is one which contains other proposition as it’s component. “Ramesh is honest” is a simple proposition. “Ramesh is honest and Dinesh is intelligent” is a compound proposition. It contains two simple propositions as it’s components.

7. Define simple statement

   A simple proposition is one which does not contain any other proposition as it’s component. A simple proposition cannot be divided into other propositions. “Ramesh is honest” is a simple proposition.

8. Define general statement

   A general proposition is a quantified statement. The categorical propositions of traditional logic are general propositions. There are four types of
general propositions, Universal Affirmative (A), Universal Negative (E), Particular Affirmative (I), Particular Negative (O).

9. Name the five compound propositions

There are five types of compound propositions. They are, Negation, Conjunction, Disjunction, Conditional statement and Bi-Conditional statement. Negation, example, ‘Ramesh is not honest’. Conjunction, example, ‘Ramesh is honest and Dinesh is intelligent’. Disjunction, example, ‘Ramesh is either intelligent or hard working’. Conditional statement, example, “If it rains, then the road will be wet”. Bi-conditional, exampl, “I will go to the cinema if and only if my friend comes with me”.

10. Name the four general propositions

There are four types of general propositions, Universal Affirmative (A), Universal Negative (E), Particular Affirmative (I), Particular Negative (O).

Universal Affirmative (A): -Example, “All men are mortal”
Universal Negative: -Example, “No men are perfect”.
Particular Affirmative: -Example, “Some men are honest”
Particular Negative: - Example, “Some men are not honest”.

11. Define truth-functional compound statement

A compound proposition is truth-functional if and only if it’s truth-value is completely determined by the truth-values of it’s component statements. There are several types of truth-functional compound statements. They are, negation, conjunction, disjunction, implication and bi-conditional

12. Give the truth-table for negation

13. State the meaning of material implication

14. Symbolize the two paradoxes of material implication

15. Comment on truth-functional operator

16. Name the four truth-functional connectives

17. Symbolize negation and conjunction

18. Symbolize disjunction and implication

19. Symbolize material equivalence.

20. Define argument form

21. Define specific form of argument

22. Define substitution instance of argument form

23. Define invalid argument form

24. Define valid argument form

25. State the principle of refutation by logical analogy
26. Symbolize disjunctive syllogism
27. Symbolize modus ponens
28. Symbolize modus tollens
29. Symbolize fallacy of affirming the consequent
30. Symbolize fallacy of denying the antecedent
31. Define statement form
32. Define specific form of statement
33. Define substitution instance of statement form
34. Define tautology
35. Define contradictory statement form
36. Define tautologous statement form
37. Define contingent statement form
38. Define Elementary valid argument form
39. Symbolize the rule of Absorption
40. Symbolize the rule of conjunction
41. Symbolize the rule of addition
42. Symbolize the rule of Hypothetical syllogism
43. Symbolize the rule of commutation
44. Symbolize the rule of distribution
45. Symbolize the rule of Association
46. Symbolize the rule of material implication
47. Symbolize the rule of logical equivalence.
48. Symbolize the rule of transposition
49. Symbolize the rule of exportation
50. State the rule of replacement
51. Comment on singular proposition
52. Comment on individual constant
53. Comment on individual variable.
54. Symbolize the singular proposition, ‘Suresh is honest’
55. Define propositional function
56. Define instantiation
57. Define quantification
58. Define singly general proposition
59. Define multiply general proposition
60. Comment on universal quantifier
61. Comment on existential quantifier.

**B-Short Answer Questions (2weightages)**

1. Distinguish between classical logic and symbolic logic

   In the history of logic, Classical logic and Symbolic logic are two important stages of development. Classical logic is also called Aristotelian logic or Ancient logic/Traditional logic, in contrast to Symbolic logic or Mathematical logic or Modern logic.

   Symbolic logic originated in connection with mathematical theory. Symbolic logic has a short history and the traditional or classical Aristotelian logic has a long one. But the difference between them is that of different stages of development. The Symbolic logic having a mathematical appearance involving heavy use of specialized technical symbols and minimum use of language, is the development and refinement of Aristotelian logic or classical logic. The Aristotelian logic contains germs of development in the manner and direction given to it by modern logicians. Classical logic and symbolic logic are interconnected. Classical logic is related to symbolic logic as embryo to adult organism.

   According to Basson and O’ Conner, “modern symbolic logic is a development of the concepts and techniques which were implicit in the work of Aristotle”. It is now generally agreed by logicians that modern symbolic logic is a development of concepts and techniques which were implicit in the work of Aristotle.

2. Explain the characteristics of symbolic logic

   A distinguished modern logician C.I. Lewis has cited three characteristics of symbolic logic:

   **1. The use of ideograms.**

   Ideograms are signs which stand directly for concepts. For example, the multiplication sign (x) or the question mark (?) are ideograms. In symbolic logic symbols which stands directly for concepts are used.

   **2. The deductive method.**

   The characteristic of this method is that from a small number of statements we can generate, by the application of a limited number of rules, an indefinite number of other new statements.

   **3. The use of variables.**

   In symbolic logic, variables have an extensive use.

   These three characteristics of symbolic logic are also characteristics of mathematics. Thus the development of symbolic logic has been bound up with
the development of mathematics. All the pioneers of symbolic logic were either mathematicians or philosophers with a training in mathematical methods.

3. Explain logical form in symbolic logic

The structure, form, or organization of a thing is constituted by the way in which its parts are put together and by the mutual relations between them. We can speak of the logical form of a statement or the set of statements constituting an argument. By doing so, the form or structure of the statement or argument is distinguished from its subject matter. In logic, only the form of argument is important, not the subject matter. That is why in symbolic logic the words which refer to the subject matter is dispensed with and replaced by variables and constants. The use of symbols in logic and logical form are closely connected. In the classical logic, one of the main uses of a good symbolic notation is to show the logical form. One of the advantages of symbolic logic over its less developed classical logic is that it has a more complete symbolic device which enables to show the logical form of arguments than the Aristotelian logic.

The reason for the logicians are interested in logical form is that logicians are interested in validity and the validity of arguments depends on their logical form. Two conditions are necessary to guarantee the truth of the conclusion of an argument. First, the evidence or premises from which deductions are made must be true. Secondly, the deductions must be valid. Of these two conditions, logic can guarantee only the second. The guarantee conferred by the validity of an argument is this:

If the premises of a valid argument are true, the conclusion is certainly true also. But where the premises are not true, the conclusion may be true or false even if the argument is valid. Thus logic does not concern directly with the factual truth of statements, even if those statements are premises or conclusions of arguments. Methods of testing the validity of various forms of arguments involve attention to the logical structure or logical form of arguments in that the validity of arguments is dependent on certain features of its logical form. To give attention to the logical form, it will be convenient to represent all arguments of a certain form by means of an appropriate symbolic notation in order that the relevant features of the structure be made clear.

4. Explain uses of symbols in symbolic logic

The greater extent to which modern logic has developed its own special technical language has made it immeasurably more powerful a tool for analysis and deduction. The special symbols of modern logic help us to show with greater clarity the logical structures of propositions and arguments whose forms may tend to be obscured by the unwieldiness of ordinary language.

1. **The use of symbols in symbolic logic helps us to bring out the features of logical importance in arguments and classify them into types.**

   Logic provide methods of testing the validity of arguments. In order to do this, arguments are to be classified into different types such that each type of argument has certain features in common with others of the same type. The
features which arguments have in common are called logical form of the argument. The traditional method of classifying arguments into types which was first invented by Aristotle involves the use of symbols. The use of variables in logic enables us to state general rules for testing the validity of arguments. Thus one important function of symbols in logic is to express the generality of the rules of logic.

2. Another important use of symbols in logic is to give conciseness and economy of expression to complicated statements which would be difficult or impossible to understand if they were expressed in ordinary language.

The situation here is comparable to the one leading to the replacement of Roman numerals by the Arabic notation. Arabic numerals are clearer and more easily comprehended than the older Roman numerals that they displaced.

3. Special symbols in logic representing logical operations facilitated the appraisal of arguments.

Technical terms in mathematics and logic are symbolized by special symbols. The drawing of inferences and the appraisal of arguments is greatly facilitated by the adoption of special logical notation.

Alfred North Whitehead one of the great contributers to the advance of symbolic logic, stated “.........by the aid of symbolism, we can make transitions in reasoning almost mechanically by the eye, which otherwise would call into play the higher faculties of the brain”.

Logic is not concerned with developing our powers of thought but with developing techniques that permit us to accomplish some tasks without having to think so much.

5. Distinguish between simple, compound and general propositions

Propositional Logic deals with arguments containing simple and compound statements. The two types of statements dealt within propositional logic are, simple and compound statements. The modern logicians have broadly classified propositions into simple and compound propositions.

A simple statement is one that does not contain any other statement as a component. A simple proposition cannot be analysed into further propositions. For example, ‘Ramesh is honest’. A simple statement cannot be further analysed into statement or statements. ‘Ramesh is honest’ does not contain any other statement as a component.

A compound statement is one that does contain another statement as a component. For example, ‘Ramesh is not honest’. ‘Ramesh and Dinesh are honest’. A compound statement can be further analysed into a statement or statements. ‘Ramesh is not honest’ can be analysed into 'Ramesh is honest' which is a statement and ‘not’ which is a word. It contains ‘Ramesh is honest’ as a component. ‘Ramesh and Dinesh are honest’ can be further analysed into ‘Ramesh is honest’ which is a statement and ‘Dinesh is honest’ which is another statement. It contains ‘Ramesh is honest’ as a component and ‘Dinesh is honest’ as another component.
There are five types of compound propositions. They are, Negation, Conjunction, Disjunction, Conditional statement and Bi-Conditional statement.

**General proposition**: A general proposition is a quantified statement. The categorical propositions of traditional logic are general propositions. There are four types of general propositions, Universal Affirmative(A), Universal Negative(E), Particular Affirmative(I), Particular Negative(O).

6. **Distinguish between weak and strong disjunction**

   Logicians recognize two kinds of disjunctions, inclusive disjunction and exclusive disjunction. A disjunction containing non-exclusive alternatives is called inclusive disjunction. Example, ‘Ramesh is either intelligent or hard working’. The sense of ‘or’ in inclusive disjunction is ‘at least one, both may be’. A disjunction containing exclusive alternatives is called exclusive disjunction. Example, ‘Today is either Wednesday or Thursday’. The sense of ‘or’ in exclusive disjunction is ‘at least one, not both’.

   The two kinds of disjunction have a part of their meanings in common. This partial common meaning, that ‘at least one’ is the whole meaning of the inclusive “or” and part of the meaning of the exclusive “or”. The common meaning, that is, the inclusive sense is represented by the Latin word “vel”. The initial letter of the word “vel” is used to represent the inclusive sense. Where p and q are any two statements whatever, their inclusive disjunction is written as “p v q”. Inclusive or weak disjunction is false only in case both of its disjuncts are false. The symbol “v” is a truth-functional connective.

7. **Distinguish between conjunction and disjunction as truth-functional compound statements**

   Conjunction is a compound proposition in which the word “and” is used to connect simple statements. Conjunction of two statements is formed by placing the word “and” between them. The two component statements of conjunction are called “conjuncts”. For example, ‘Ramesh is honest and Dinesh is intelligent’. ‘Ramesh is honest’ is the first conjunct and ‘Dinesh is intelligent’ is the second conjunct. To have a unique symbol whose only function is to connect statements conjunctively, the dot “.” Symbol is used for conjunction. The above conjunction is represented as “Ramesh is honest . Dinesh is intelligent”. It is symbolized as “R.D” where ‘R’ represents “Ramesh is honest, ‘D’ represents ‘Dinesh is intelligent”. More generally, where p and q are any two statements whatever their conjunction is symbolized as ‘p . q’.

   In conjunction, if both its conjuncts are true, the conjunction is true, otherwise it is false. For this reason a conjunction is said to be a truth-functional compound statement. The dot “ . ” symbol is a truth-functional connective.

   The truth-table for conjunction can be represented as follows:
As shown by the truth-table defining the dot symbol, a conjunction is true if and only if both of its conjuncts are true.

Disjunction is a compound proposition in which the simple propositions are connected by the word ‘or’ or the phrase ‘either...... or’. Disjunction or alternation of two statements is formed by inserting the word “or” or the phrase “either....or” between them. The two component statements are called “disjuncts” or “alternatives”. For example, ‘A or B’, ‘Either A or B’.

There are two types of disjunction, Inclusive disjunction and Exclusive disjunction. The inclusive “or” has the sense of “at least one, both may be”. This sense of the word “or” is also called “weak”. For example, ‘Arun is either intelligent or honest’. The meaning is that Arun is intelligent only, or honest only, or both intelligent and honest. The exclusive “or” has the sense of “at least one, not both”. This sense of the word “or” is also called “strong”. For example, ‘Today is Sunday or Monday’. The meaning is that today is Sunday or Monday but not both Sunday and Monday. The two kinds of disjunction have a part of their meanings in common. This partial common meaning, that ‘at least one’ is the whole meaning of the inclusive “or” and part of the meaning of the exclusive “or”. The common meaning, that is, the inclusive sense is represented by the Latin word “vel”. The initial letter of the word “vel” is used to represent the inclusive sense. Where p and q are any two statements whatever, their inclusive disjunction is written as “p v q”. Inclusive or weak disjunction is false only in case both of it’s disjuncts are false. The symbol “v” is a truth-functional connective.

The truth-table for disjunction is as follows:

<table>
<thead>
<tr>
<th>P</th>
<th>q</th>
<th>P v q</th>
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</thead>
<tbody>
<tr>
<td>T</td>
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Implication is a compound proposition in which the simple statements are connected by the phrase ‘if ...... then’. For example, “If it rains, then the road will be wet”. The part of proposition which lies in between ‘if’ and ‘then’ is called the antecedent or implicants. The part of proposition which follows the word ‘then’ is called the consequent or implicate. The general form of an implicative
A conditional statement asserts that in any case in which its antecedent is true, its consequent is true also. It does not assert that its antecedent is true, but only that if its antecedent is true, its consequent is true also.

There are different types of implication each of which asserts a different sense of “if ….. then”.

In the statement ‘If all humans are mortal and Socrates is a human, then Socrates is mortal’, the consequent follows logically from its antecedent.

In the statement ‘If Ramesh is a bachelor, then Ramesh is unmarried’, the consequent follows from its antecedent by the very definition of the term ‘bachelor’, which means unmarried man.

In the statement, ‘If blue litmus paper is placed in acid, then it will turn red’, the relation between antecedent and consequent is causal. In the example, ‘If Indian cricket team loses its match against Australia, then I will eat my hat’, the decision of the speaker is reported.

The above four types of conditional statements are all conditional statements. The common meaning, that is part of the meaning of all four different types of implication, is to be found out. Conditional statement does not assert that its antecedent is true, but asserts that if the antecedent is true, then its consequent is also true. Conditional statement is false if its antecedent is true and consequent is false. That is, it cannot be the case that the antecedent is true and consequent is false. Any conditional statement “If $p$ then $q$” in case the conjunction “ $p \lor \neg q$ ” is known to be false. For a conditional to be true the conjunction “ $p \land q$ ” must be false. That is, it’s negation “ $p \land \neg q$ ” must be true. Thus, “ $p \land q$ ” is regarded the common meaning that is part of the meaning of all four different types of implication symbolized as “ If $p$, then $q$”. Every conditional statement means to deny that its antecedent is true and it’s consequent false. This common partial meaning of the “if….then” is symbolized as “ $\iff$ ” called “ horseshoe “. “ If $p$ then $q$ “ is symbolized as $p \iff q$ as an abbreviation for “ $p \land \neg q$ ”. The symbol “ $\iff$ ” can be regarded as representing another kind of implication called “material implication”. No real connection between antecedent and consequent is suggested by material implication. It asserts that “it is not the case that the antecedent is true and the consequent is false”.

The material implication symbol is a truth-functional connective.

The truth-table for implication is as follows:
If it rains, then the road will be wet
If it rains, the road will be wet
The road will be wet if it rains
It rains implies that the road will be wet.
The road being wet is implied by it rains.

All these conditional statements are symbolized as $R \supset W$

Where $p$ and $q$ are any two statements implication between any two statements is symbolized as “$p \supset q$“. This can also be read as “$p$ is a sufficient condition for $q$“, “$q$ is a necessary condition for $p$“, “$p$ only if $q$“.

9. **Distinguish between material implication and material equivalence.**

Implication:- Conditional statement does not assert that it’s antecedent is true, but asserts that if the antecedent is true, then it’s consequent is also true. Conditional statement is false if it’s antecedent is true and consequent is false. That is, it cannot be the case that the antecedent is true and consequent false. Any conditional statement “If $p$ then $q$” in case the conjunction “$p \land q$” is known to be is false. For a conditional to be true the conjunction “$p \land q$“ must be false. That is, it’s negation ($p \land \neg q$) must be true. Thus, ($p \land \neg q$) is regarded the common meaning that is part of the meaning of all four different types of implication symbolized as “If $p$, then $q$”. Every conditional statement means to deny that it’s antecedent is true and it’s consequent false. This common partial meaning of the “if....then” is symbolized as “$\supset$“ called “horseshoe“. “If $p$ then $q$“ is symbolized as $p \supset q$ as an abbreviation for ($p \land \neg q$).

The symbol “$\supset$“ can be regarded as representing another kind of implication called “material implication”. No real connection between antecedent and consequent is suggested by material implication. It asserts that “it is not the case that the antecedent is true and the consequent is false”.

The material implication symbol is a truth-functional connective.
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All these conditional statements are symbolized as $R \supset W$

Where $p$ and $q$ are any two statements implication between any two statements is symbolized as $p \supset q$. This can also be read as “$p$ is a sufficient condition for $q$”, “$q$ is a necessary condition for $p$”, “$p$ only if $q$.”

**Bi-conditional proposition**: Bi-conditional proposition is a compound proposition in which the simple statements are connected by the phrase ‘if and only if’. For example, “I will go to the cinema if and only if my friend comes with me”. Bi-conditional proposition is also called ‘material equivalence’. When two statements are combined by using the phrase “if and only if”, the resulting compound statement is called bi-conditional statement or Material Equivalence. The phrase “if...then” is used to express sufficient condition, “only...if” phrase is used to express necessary condition. The phrase “if and only if” is used to express both sufficient and necessary condition. The above example can also be stated as, ‘I will go to the cinema if my friend comes with me and, I will go to the cinema only if my friend comes with me’. The phrase “If and only if” is symbolized as $\equiv$.

Where $p$ and $q$ are any two statements whatever, their material equivalence is represented as $P \equiv q$. When two statements are materially equivalent, they materially imply each other.

Two statements are said to be materially equivalent when they have the same truth-value. The symbol $\equiv$ is a truth-functional connective. The truth-table for biconditional or material equivalence is as follows:

<table>
<thead>
<tr>
<th>P</th>
<th>q</th>
<th>$P \equiv q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

Negation of a statement is formed by the insertion of the word “not” in the original statement or by prefixing to it the phrase ‘it is not the case that’ or ‘it is false that’. The symbol “¬” called “curl” or “tilde” is used to form the negation of a statement. Thus the above example is represented as ¬R where R symbolizes ‘Ramesh is honest’. More generally, where p is any statement whatever, it’s negation is symbolized as ‘¬p’.

Negation is a truth-functional compound statement and the curl “¬” is a truth-functional operator.

The negation of any true statement is false, and the negation of any false statement is true. The truth-table for Negation can be represented as follows:

\[
\begin{array}{c|c}
P & ¬p \\
T & F \\
F & T \\
\end{array}
\]

where p and q are any two statements whatever their conjunction is symbolized as ‘p . q’.

In conjunction, if both it’s conjuncts are true, the conjunction is true, otherwise it is false. For this reason a conjunction is said to be a truth-functional compound statement. The dot “.” symbol is a truth-functional connective.

The truth-table for conjunction can be represented as follows:

\[
\begin{array}{c|c|c}
P & q & p . q \\
T & T & T \\
T & F & F \\
F & T & F \\
F & F & F \\
\end{array}
\]

As shown by the truth-table defining the dot symbol, a conjunction is true if and only if both of it’s conjuncts are true.

Where p and q are any two statements whatever, their inclusive disjunction is written as “p v q”. Inclusive or weak disjunction is false only in case both of it’s disjuncts are false. The symbol “v” is a truth-functional connective.

The truth-table for disjunction is as follows:

\[
\begin{array}{c|c|c}
P & q & p v q \\
T & T & T \\
T & F & T \\
F & T & T \\
F & F & F \\
\end{array}
\]
The material implication symbol is a truth-functional connective. The possible combinations of truth-values for implication is represented as follows:

Where p is true and q is true, \( p \supset q \) is true.
Where p is true and q is false, \( p \supset q \) is false.
Where p is false and q is true, \( p \supset q \) is true.
Where p is false and q is false, \( p \supset q \) is true.

The truth-table for implication is as follows:

<table>
<thead>
<tr>
<th>P</th>
<th>q</th>
<th>p ( \supset ) q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

Where \( p \) and \( q \) are any two statements whatever, their material equivalence is represented as ‘\( P \equiv q \)’. When two statements are materially equivalent, they materially imply each other. Two statements are said to be materially equivalent when they have the same truth-value. The symbol ‘\( \equiv \)’ is a truth-functional connective. The truth-table for biconditional or material equivalence is as follows:

<table>
<thead>
<tr>
<th>P</th>
<th>q</th>
<th>p ( \equiv ) q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

11. Explain punctuation in symbolic logic
12. Distinguish between argument and argument form
14. Explain the validity of modus ponens by truth-table method by truth-table method
15. Explain the validity of modus tollens by truth-table method
16. Explain the fallacy of affirming the consequent by truth-table method
17. Explain the fallacy of denying the antecedent by truth-table method
18. Explain tautologous statement form
19. Explain contradictory statement form
20. Explain contingent statement form
21. Explain logical equivalence
22. Explain De Morgan’s theorem
23. Explain the relation between Arguments, conditional statements, and tautologies
24. State the first nine rules of inference.
25. State the last ten rules of inference.
26. Explain the shorter method of proving invalidity by assigning truth-values
27. Explain the need of predicate logic
28. Distinguish between instantiation and quantification.
29. Explain symbolization of subject-predicate propositions, A and E
30. Explain symbolization of subject-predicate propositions, I and O
31. Explain the relation between universal and existential quantification
32. Explain logical form in symbolic logic
33. Explain uses of symbols in symbolic logic
34. Distinguish between propositional logic and predicate logic.

C-Essay Questions
1. Explain the relation between classical logic and symbolic logic.
2. Explain the modern classification of propositions
4. Explain the method of proving validity or invalidity of arguments based on the principle of refutation by logical analogy (truth-table method)
5. State the relation between statement and statement form. Explain the different statement forms.
6. Explain formal proof of validity
7. Explain the rule of replacement
8. Explain quantification theory

Reference Books:
1) I. M. Copi and Carl Cohen, Introduction to Logic
   Prentice-Hall of India, New Delhi
2) P. Balasubramanian, Symbolic Logic
3) A. H. Basson and D.J. G. Connor,
   Introduction to Symbolic Logic
   Oxford University Press, Delhi

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