FOURTH SEMESTER M.Sc DEGREE (MATHEMATICS) EXAMINATION, JUNE 2012
(CUCSS-PG-2010)
MT4E01 : COMMUTATIVE ALGEBRA

MODEL QUESTION PAPER

Time: 3 hrs. Max. Weightage: 36

PART - A

Short Answer Questions (Answer all questions) Each question has weightage 1

1. Define the radical of an ideal in a ring
2. In Z what is the radical of the ideal <4>
3. Define a local ring.
4. Give an example of coprime ideals in Z
5. Define a faithful A-module
6. Define a finitely generated A-module
7. Define a multiplicatively closed set in a ring A. Give an example.
8. Give an example of a primary ideal in Z which is not a prime ideal.
9. What is ascending chain condition.
10. Give an example of a ring which is Artin but not Noetherian.
11. Define irreducible ideal in a ring.
12. Define the dimension of a ring A.
13. What is the integral closure of Z in Q.
14. Let B be an integral domain and K its field of fractions. What is the condition that B is a valuation ring of K.

(14 x 1=14)

PART - B

Paragraph Questions (Answer any 7 questions).Each question has weightage 2

15. Prove that the nilradical of a ring A is the intersection of all prime ideals in A.
16. Let $P_1, P_2, \ldots P_n$ be prime ideals and let a be an ideal such that a $\subseteq \bigcup_{i=1}^{n} P_i$ then show that a $\subseteq P_i$ for some i = 1,2, …. n.
17. Define the tensor product of two A - modules M and N. Prove that it is unique upto isomorphism.
18. If m and n are coprime, show that $Z_m \otimes Z_n = 0$
19. If $M' \xrightarrow{f} M \xrightarrow{g} M''$ is an eract sequence of A – modules then show that $S^{-1}M' \xrightarrow{s^{-1}f} S^{-1}M \xrightarrow{s^{-1}g} s^{-1}M''$ is an exact sequence of S^{-1}A modules.
20. Give an example of a primary ideal which is not a power of a prime ideal.
21. Prove that M is a Noetherian A module if and only if every sub module of M is finitely generated.
22. Show that in a Noetherian ring every irreducible ideal $\neq \langle 1 \rangle$ is primary.
23. Show that an Artin ring has only finitely many maximal ideals.
24. Show that any unique factorization domain is integrally closed.

(7 x 2 = 14)

PART – C

Essay type questions (Answer any two questions). Each question has weightage 4

25. State and prove Nakayama's lemrna.
26. State and prove the first uniqueness theorem in primary decomposition.
27. State and prove the Gowing up theorem.
28. State and prove the structure theorem for Artin rings.

(2 x 4 = 8)