THIRD SEMESTER MSc DEGREE EXAMINATION

MODEL QUESTION PAPER

MT3C14 - LINEAR PROGRAMMING AND ITS APPLICATIONS

Time: 3 Hours MaximumWeightage: 36

Part A

(Short Answer Type Questions)

Answer All Questions. Each question has weightage one

1. Find the convex hull of the set of points $(1,0,0),(0,1,0),(0,0,1)$ in E_3 .
2. Express the point $(1,\frac{1}{2})$ as convex linear combinations of the points $X_1=(0,0), X_2=(2,0), X_3=(1,1)$
3. Find $\nabla f(X)$ and $H(X)$ for $f(X)=x_1^3+2x_2^3+3x_1x_2x_3+x_3^2$.
4. Prove that $f(x)=x^2, x\in R$ is a convex function.
5. Find the maximum number of basic solutions and all the basic feasible solutions of the LPP Maximize $-6x+2x_2-4x_3+5x_4$ subject to $4x_1-x_2+2x_3+3x_4\le 1, x_2+4x_3-2x_4\le 2, x_1,x_2,x_3,x_4\ge 0$
6. Write both the primal and dual LPP corresponding to the following payoff matrix $\begin{bmatrix}0&1&2\\2&0&1\\1&2&0\end{bmatrix}$
7. Give examples of convex sets (i) with no vertex (ii) one vertex only.
8. Find the unit vector in the direction of the steepest ascent of $f(X)=x_1^2+2x_1x_2+x_1x_3+x_2x_4+x_4^2$ at the point (1, 0,-1, 1).
9. Determine the range of value of p and q that will make the pay off element a_{22} a saddle point for the game whose pay off matrix (a_{ij}) is given as. $\begin{bmatrix}2&4&5\\10&7&q\\4&p&8\end{bmatrix}$.
10. Find the optimal objective value of the LPP by only inspecting its dual Minimize $z=10x_1+4x_2+5x_3+x_4$ subject to $5x_1-7x_2+3x_3+\frac{1}{2}x_4\ge 150$. $x_1,x_2,x_3,x_4\ge 0$.
11. How do you recognize optimality in the simplex method?
12. Define the terms 1) basic feasible solution 2) degenerate basic solution.
13. Give an example of a balanced transportation problem.
14. Define separating and supporting hyper planes of a convex set.

(14 x 1=14)

Part B

(Paragraph Type Questions)

Answer any seven Questions. Each question has a weightage two

15. Find the relative maxima minima and saddle points if any of $f(X)=x_1^3+x_2^3-3x_1-12x_2+25$
16. Prove that every point of $[S]$ can be expressed as s convex linear combination of at most (n+1) points of $S\subseteq E_n$.
17. Prove that a vertex of S_F is a basic feasible solution.
18. Solve the following LPP by solving the dual by graphical method and using complementary slackness conditions Maximize $f=3x_1+2x_2+x_3+4x_4$ subject to $2x_1+2x_2+x_3+3x_4\le 20, 3x_1+x_2+2x_3+2x_4\le 20, x_1,x_2,x_3,x_4\ge 0$
19. Prove that the transportation problem has a triangular basis.

20. Prove the following. An optimal solution of Min $f(X) = CX$ subject to $X \in T_F$ is an optimal solution of Minimize $f(X) = CX$ subject to $X \in [T_F]$.

21. Use the notion of dominance to simplify the following payoff matrix and then solve $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 0 & 2 & 1 \\ 4 & 3 & 2 & 3 & 2 \\ 4 & 3 & 4 & -1 & 2 \end{bmatrix}$.

22. Solve the LPP by graphical method Maximize $5x_1 - x_2$ subject to $x_1 + x_2 \geq 2, x_1 + 2x_2 \leq 2, 2x_1 + x_2 \leq 2, x_1, x_2 \geq 0$.

23. Find the minimum assignment cost of the following problem

	$M1$	$M2$	$M3$	$M4$
A	5	2	–	5
B	7	3	2	4
C	9	–	5	3
D	7	7	6	2

24. Prove that intersection of two convex sets is a convex set. (7 x 2 =14)

Part C

(Essay Type Questions)

Answer any two Questions. Each question has a weightage four.

25. a) Prove that the dual of the dual of an LPP is primal.

b) Solve the LPP using simplex method

Maximize $5x_1 + 3x_2$ subject to $4x_1 + 5x_2 \leq 10, 5x_1 + 2x_2 \leq 10, 3x_1 + 8x_2 \leq 12, x_1, x_2 \geq 0$

26. a)Describe the cutting plane method

b) Solve the following transportation problem to find the minimum cost

	D	E	F	G	H	
A	5	8	6	6	3	800
B	4	7	7	6	5	500
C	8	4	6	6	4	900
	400	400	500	400	800	

27. a).State and prove fundamental theorem on matrix games.

b).solve graphically the game whose payoff matrix is $\begin{bmatrix} 2 & 7 \\ 3 & 5 \\ 11 & 2 \end{bmatrix}$.

28. a)Describe the caterer problem

b) Solve the following LPP by the dual simplex method. Minimize $2x_1 + 3x_2$ subject to

$$2x_1 + 3x_2 \leq 30$$
$$x_1 + 2x_2 \geq 10$$
$$x_1, x_2 \geq 0$$

(2 x 4 =8)