FOURTH SEMESTER M.Sc DEGREE (MATHEMATICS) EXAMINATION, JUNE 2012
(CUCSS-PG-2010)
MT4E05 : OPERATIONS RESEARCH

MODEL QUESTION PAPER

Time: 3 hrs. Max. Weightage: 36

PART – A

(Short Answer Type Questions)
Answer all questions. Each question is of weightage 1.

1. Explain the concept of connectedness in directed graph.
2. Define 'cut' in a graph of the maximum flow problem. Explain its role in the maximum flow problem.
3. Explain the method of sensitivity analysis for the changes in the cost coefficients c_j.
4. Distinguish between sensitivity analysis and parametric linear programming.
5. Find graphically the minimum of $(x_1-3)^2 + (x_2-3)^2$ subject to $(x_1-1)(x_2-1) \le 1, x_1 + x_2 \ge 6, x_1, x_2 \ge 0$.
6. Write down the Kuhn-Tucker conditions for the problem: Minimise $(x_1 + 1)^2 + (x_2 - 2)^2$ subject to $x_1-2 \le 0, x_2-1 \le 0, x_1 \ge 0, x_2 \ge 0$.
7. What is quadratic programming problem in Operations Research?
8. White down the standard from of a geometric programming problem.
9. Convert the following problem in to the form of a geometric programming problem: Find the dimensions of the rectangle of maximum area inscribed in a circle of radius r.
10. Briefly explain the method of Dynamic programming to solve the minimum path problem.
11. Use Dynamic programming method to Maximise u_1u_2 subject to $u_1 + u_2 = 7, u_1,u_2 \ge 0$.
12. Define unimodal function. Give an example of a real valued function which is unimodal in [0,10].
13. Describe the Rosenbrock algorithms to locate the minimum of a function.
14. Explain briefly the general method of axial directions.

(14x1=14)

PART – B

(Paragraph Type Questions)
Answer any 7 questions. Each question is of weightage 2

15. Find the minimum spanning tree in the following undirected graph

Are	(1,2)	(1,3)	(1,4)	(2,3)	(2,8)	(2,10)	(3,4)	(3,8)	(4,5)	(4,6)	(4,8)	(5,6)	(5,7)
Length	7	4	8	3	9	14	4	10	15	12	10	4	1

(6,7)	(6,8)	(6,9)	(7,9)	(8,9)	(8,10)	(9,10)
2	20	16	18	3	4	6

16. Describe the problem of scheduling sequential activities. Explain how it is converted to the problem of minimum path.

17. The following table gives an optimal solution to the problem minimize $3x_1 + 5x_2 + 2x_3$ Subject to $-x_1 + 2x_2 + 2x_3 \ge 3, x_1 + 2x_2 + x_3 \ge 2, 2x_1 + x_2 - 2x_3 \le 4, x_1, x_2, x_3 \ge 0,$

Basic	13	P_1	P_2	P_3	P_4	P_5	P_6
x_3	2	1	2	1		-1	
x_4	1	3	2		1	-2	
x_6	8	4	5			-2	1
f	-4	1	1			2	

Where x_4 and x_6 are the slack variables introduced in the first and third constraints respectively. Do sensitivity analysis if the first column of the coefficient matrix is changed to (1,2,-2).

18. Explain the method of parametric linear programming in the case of variations in b_i.
19. Let X_o be a solution of the convex programming problem minimize f(X), subject to $g_i(X) \leq 0$, 1=1,….,m, f(X) and g(X) are convex functions. Prove that if G(X) = $(g_i(X), \ldots, g_n(X))$ and the set of points X such that $G(X) < 0$, is non-empty, then there exists a vector $Y_o \geq 0$ such that $f(X) + Y_o' G(X) \geq f(X_o)$.
20. Verify that Kuhn – Tucker theory fails to give solution of the problem: Minimise $x_1^2 + x_2^2$, subject to $(x_1-1)^3 - x_2^2 \geq 0$; $x_1, x_2 \geq 0$.
21. Use dynamic programming method to maximize $u_1^2 + u_2^2 + u_3^2$ subject to $u_1u_2u_3 \leq 6$ where u_1, u_2 and u_3 are positive integers.
22. Prove that in a serial two-stage minimization problem if (1) the objective function ϕ_2 is a separable function of stage returns $f_1(X_1,U_1)$ and $f_2(X_2,U_2)$, and (2) ϕ_2 is monotonic non decreasing function of f_1 for every feasible value of f_2, then the problem is decomposable.
23. Describe the method of quadratic interpolation.
24. Explain the method of steepest descend in multidimensional search.

PART – C

(Essay Type Questions)

Answer any two questions. Each question is weightage 4.

25. Maximize $f(x) = -5x_1 + 13x_2 + 5x_3$ subject to $12x_1 + 10x_2 + 4x_3 \leq 90$, $-x_1 + 3x_2 + x_3 \leq 20$, $x_1, x_2, x_3 \geq 0$.

    Use sensitivity analysis to investigate the effects on the optimal solution if a new constraint $2x_1 + 5x_2 + 3x_3 \leq 50$, is introduced.
26. Use the method of quadratic programming to solve Minimize $f(X) = -x_1 - x_2 - x_3 + \frac{1}{2}(x_1^2 + x_2^2 + x_3^2)$

    Subject to $x_1 + x_2 + x_3 - 1 \leq 0$

    $$4x_1 + 2x_2 - \frac{7}{3} \leq 0$$

    $$x_1, x_2, x_3 \geq 0.$$
27. Use geometric programming to find the minimum of $\frac{c_1}{x_1 x_2^{1/2} x_3} + c_2x_1x_3 + c_3x_1x_2x_3$

    subject to $\frac{c_4}{x_1^2 x_2^2} + \frac{c_5 x_2^{1/2}}{x_3} \leq 1$, $c_i>0$, $x_j>0$, i=1,2,3,4,5; j=1,2,3.
28. Describe the Fibonacci search plan in one dimensional search

(2 x 4 = 8)