FOURTH SEMESTER M.Sc DEGREE (MATHEMATICS) EXAMINATION,
JUNE 2012
(CUCSS-PG-2010)

MT4E06 : PROBABILITY THEORY

MODEL QUESTION PAPER

Time: 3 hrs. Max. Weightage: 36

PART A

(Short Answer Type Questions)
Answer all the questions. Each question has weightage 1

1. Prove that a $\sigma - field$ is a monotone field.
2. If $A_n \to A$, show that $A_n^c \to A^c$.
3. List the set of all events in the experiment of tossing three coins simultaneously.
4. Define random variable. Give an example.
5. Explain discrete probability space.
6. Define signed measure with an example.
7. State Jordan decomposition theorem.
8. Prove that if X≥ 0 a.s. then EX ≥ 0 .
9. If $X_n \xrightarrow{p} X$ and $X_n \xrightarrow{p} X'$, then prove that X and X' are equivalent.
10. Prove that for a ch.fn. φ, $\text{Re}(1-\phi(u) \geq \frac{1}{4}\text{Re}(1-\phi(2u))$.
11. Prove that if $F_n \xrightarrow{w} F$, then F is unique.
12. Give an example of events A,B,C such that {A, B, C} are pair-wise independent, but {A, B, C } are not mutually independent.
13. Prove that { S_n , n$\geq$1} converges in q.m. if and only if $\sum \sigma_k^2 < \infty$.
14. Define tail equivalence and convergence equivalence of events.

(14 x 1 = 14)

PART B

(Paragraph Type Questions)
Answer any seven questions. Each question has weightage 2

15. Show that the intersection of an arbitrary collection of $\sigma - fields$ is again a $\sigma - field$.

16. If X and Y are random variables, then show that min(X,Y) is also a random variable.
17. Prove that probability function is a continuous function.
18. If {A_n }is a monotone sequence of events, then show that $\lim_{n\to\infty} P(A_n) = P(\lim_{n\to\infty} A_n)$.
19. Find the value of k for which $f(x) = \frac{kx^2}{e^x}$, x≥0 is a probability density function.
20. If X and Y are two simple random variables and a and b are real numbers, then show that E(aX+bY) =aE(X)+bE(Y).
21. Distinguish between pair wise independence and mutual independence of events.
22. Show that almost sure convergence implies convergence in probability.
23. Prove that Borel functions of independent random variables are independent.
24. State a set of conditions under which the weak law of large numbers holds for a sequence of independent random variables not necessarily identically distributed.

(7 x 2 = 14)

PART C

(Essay Type Questions)
Answer any two questions-Each question has weightage 4

25. Show that a non-negative random variable X can be expressed as the limit of a sequence of non-decreasing non-negative simple random variables.
26. State and establish Jordan decomposition of a distribution function.
27. State and prove the inversion theorem for characteristic function.
28. State and prove Liapounov’s theorem.

(2 X 4 = 8)