**FOURTH SEMESTER M.Sc. DEGREE EXAMINATION**
**(CUCSS)**
**MT4C16 DIFFERENTIAL GEOMETRY**
**MODEL QUESTION PAPER**

**Time: 3 hours** **Maximum Weightage: 36**

## Part A

*Answer **all** the questions*
*Each question carries **1** weightage*

1. Compute $\nabla_v f$ where $f:\mathbb{R}^3 \to \mathbb{R}$ is given by $f(x_1, x_2, x_3) = x_1x_2x_3^2$ and $v = (1,1,1,a,b,c)$
2. Is it true that parallel transport along a path, a linear isometry?
3. Define directional derivative of a smooth vector field on a subset of $\mathbb{R}^{n+1}$.
4. Define level set of a function f from a subset of $\mathbb{R}^{n+1}$ to $\mathbb{R}$ at a height c in$\mathbb{R}$. Also sketch the level set $f^{-1}(0)$ where $f(x_1, x_2) = x_1^2 - x_2^2$.
5. Show that if $\alpha : I \to \mathbb{R}^{n+1}$ is a parameterized curve with constant speed, then $\ddot{\alpha}(t) \perp \dot{\alpha}(t)$, for all t in I.
6. Let $\alpha(t) = (x(t), y(t))$, $t \in I$ be a local parametrization of the oriented plane curve C. Show that $\kappa \circ \alpha = \frac{x'y''-y'x''}{(x'^2+y'^2)^{3/2}}$
7. Define a vector field on a subset of $\mathbb{R}^{n+1}$.
8. Explain Gauss-Kronecker curvature and mean curvature of an n-surface at a point in it.
9. Find velocity and acceleration of the parametrized curve $\alpha(t) = (\cos t, \sin t, 2\cos t, 2\sin t)$
10. Prove or disprove: Geodesics are having constant speed.
11. Find two orientations on the sphere $x_1^2 + x_2^2 + \cdots\cdots + x_{n+1}^2 = r^2$, $r > 0$
12. Define curvature of a 1-surface C in $\mathbb{R}^2$ at a point p in C.
13. Let S be a connected n-surface in $\mathbb{R}^{n+1}$ and let $g : S \to \mathbb{R}$ be smooth. Suppose g can take only the values +1 and −1. Prove that g is constant.
14. If a parametrized curve α defined on I is of unit speed, prove that the length of α, $l(\alpha) = l(I)$

**(14×1 = 14 weightage)**

## Part B

*Answer any **seven** questions*
*Each question carries **2** weightage*

15. Show that the Weingarten map of an n-surface S at a point p in S is self adjoint.
16. Prove that local parametrization of plane curves are unique up to reparametrization.
17. Write a short note on Levi-Civita parallelism and its properties.
18. Find all geodesics on the sphere $S^2$.
19. State and prove Lagrange's multiplier theorem.
20. Show by an example that for an n-surface, the curvature at any point in any tangent direction can be same.

21. For any 1-form ω on a subset U of $\mathbb{R}^{n+1}$, show that there exist functions $f_i$ : U → I such that ω = $\sum_{i=1}^{n+1} f_i \, dx_i$.
22. Let C be a connected oriented plane curve and let β : I → C be a unit speed global parametrization of C. Then prove that β is either one-one or periodic.
23. Show that Mobius band is an unorientable 2-surface.
24. Let V be a finite dimensional vector space with dot product. Let L: V → W be a self adjoint operator on V. Then show that there exists an orthonormal basis of V consisting of Eigen vectors of L.

**(7×2 = 14 weightage)**

Part C

*Answer any **two** questions*
*Each question carries **4** weightage*

25. Let g : $\mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x_1, x_2) = ax_1^2 + 2bx_1x_2 + cx_2^2$. Prove that the maximum and minimum values of g on the unit circle $x_1^2 + x_2^2 = 1$ are given by the Eigen values of the matrix $\begin{bmatrix} a & b \\ b & c \end{bmatrix}$.
26. Let S be an n-surface in $\mathbb{R}^{n+1}$, let $\mathbb{X}$ be a smooth tangent vector field on S and let p ϵ S. Then show that there exist an open interval I containing 0 and a parametrized curve α : I → S such that (1) α(0) = p
    (2) $\dot{\alpha}(t) = \mathbb{X}(\alpha(t))$, for all t∈I
    (3) If : $\bar{I}$ →S is a parametrized curve satisfying (1) and (2), then $\bar{I}$ ⊂ I and for all t ∈ $\bar{I}$, $\beta(t) = \alpha(t)$.
27. Find Gauss Kronecker curvature of the ellipsoid $\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1$ oriented by its outward normal.
28. Let φ : U → $\mathbb{R}^{n+1}$ be a parametrized n-surface in $\mathbb{R}^{n+1}$ and let p ϵ U. Then show that there exist an open set $U_1$ ⊂ U about p such that $\varphi(U_1)$ is an n-surface in $\mathbb{R}^{n+1}$.

**(2×4 = 8 weightage)**

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