FOURTH SEMESTER M.Sc DEGREE (MATHEMATICS) EXAMINATION, JUNE 2012
(CUCSS-PG-2010)
MT4C15 FUNCTIONAL ANALYSIS II

MODEL QUESTION PAPER

Time: 3 hrs. Max. Weightage: 36

PART – A

Short Answer Questions
Answer all questions. Each question carries 1 weightage.

1. Prove or disprove: A closed map from a metric space to a metric space is continuous.
2. Prove or disprove: If X is a Banach space with a norm $\|\ \|$, then any norm on X comparable to $\|\ \|$ is complete.
3. Prove that a bounded linear operator on a Banach space is invertible if and only if it is bijective
4. If X is a finite dimensional normed linear space, prove that X is linearly homeomorphic to its dual X'.
5. Prove that the set of all invertible operators on a Banach space X is an open subset of BL (X).
6. Prove that the dual of a reflexive normed space is reflexive.
7. Prove that if F is a nonempty closed subspace of a Hilbert space H, then the orthogonal complement of the orthogonal complement of F is F.
8. Prove or disprove : Every finite dimensional normed space is reflexive.
9. Prove or disprove : A sequence in a Hilbert space converges if and only if it is weak convergent.
10. Give an example of an inner product space and bounded linear operator A on it such that the adjoint of A does not exist. Indicate a proof of your claim.
11. Give an example of a self adjoint operator on K^2. Substantiate your claim.
12. Define the eigen spectrum of a bounded linear operator on a normed space X and show by an example that it may be different from the spectrum.
13. Prove that a bounded linear operator on a finite dimensional Hilbert space is compact.
14. Give an example of a compact operator whose range is not finite dimensional. Indicate a proof of your claim.

(14 x 1=14)

PART – B

Paragraph Type Questions
Answer any seven questions. Each question carries 2 weightage

15. Prove that a linear open map from a normed linear space X to a normed linear space Y is surjective.
16. Show by an example that the open mapping theorem may fail when the domain is not complete.
17. If A is a bounded linear operator on a normed space of finite rank, prove that the eigen spectrum, the approximate eigen spectrum and the spectrum of A are equal.

18. If A is bounded linear operator on a Banach space X and $\|A^P\| < 1$ for some positive integer p, prove that the bounded operator I – A is invertible.

19. Let X be a uniformly convex normed space and (x_n) a sequence in X such that $\|x_n\|$ converges to 1 and $\|x_n+x_m\|$ converges to 2 as n and m tends to infinity. Then prove that (x_n) is a Cauchy sequence.

20. If (x_n) is a bounded sequence in a Hilbert space, prove that it has a weak convergent subsequence.

21 Prove that a Hilbert space is reflexive

22. Prove that for a continuous linear functional on a subspace of a Hilbert space H, there exists a unique Hahn – Banach extension.

23. If A is a bounded linear operator on a Hilbert space and A_n is a compact operator for every positive inter n such that $\| A_n - A\|$ converges to 0, then prove that A is compact.

24. Let H be a finite dimensional Hilbert space over K and A is a bounded linear operator on H. Suppose that K = C and A is normal or K = R and A is self adjoint. Then prove that there exists an orthonormal basis for H consisting of eigen vectors of A.

(7 x 2 =14)

PART – C

Essay Type Questions
Answer any two questions. Each question carries 4 weightage

25. State and prove the closed graph theorem.

26. State and prove the Riesz representation theorem.

27. a) State and prove the generalized Schwarz inequality.
b) Prove that the adjoint of a compact operator on a Hilbert Space H is compact.

28. a) Determine the dual of l^p when $1 \le p \le \infty$.
b) Determine the dual of c_{00} with the norm $\| \|_p$.

(2 x 4 = 8)